International Trade

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Foreign Trade Theory as a Part of Economics

Foreign trade = total of all economic activities across national borders.
Partners in international trade are usually individual economic agents (households, firms, banks), sometimes also nations (cf. foreign trade monopolies) and central banks.

Categories of foreign trade relationships

- trade in goods and services
- international transfers
- credit and capital transactions
- international migration

There are (even though minor) differences between foreign and domestic economic activities.

- Classical: national mobility of factors, internationally immobile. ***But.*** Also national barriers to mobility (e.g., North-South divide, East-West-Germany); international mobility cannot be denied (foreign workers; foreign direct investments (FDI))
- Recently emphasized features are political barriers to trade, differences in national trade policies, spatial distances including transport cost. ***But.*** Transport costs do exist also on national or even regional levels.
- Different currencies. ***But.*** For fixed exchange rates and free convertibility there is hardly no difference compared to a single currency ($\rightarrow$ euro area).

Foreign trade theory

- applies general economic theories to processes of international trade.
- has to explain the fundamental, theoretical aspects of international trade relationships.
- includes theory of the household (consumption), theory of the firm (production), price —, monetary —, allocation —, employment —, growth — etc. (heterogeneous body of theories, especially coexistence of micro- and macroeconomics).

This course follows a universally microeconomic approach!

Distinguish real and monetary trade theory.

Real Foreign Trade Theory

Real foreign trade theory is attended to explain the commodity related fundamentals of international trade relations.
Exemplary problems to be explained

- determinants for the *quantitative pattern* of foreign trade
- determinants for the *price ratio* between export and import goods ($\rightarrow$ terms of trade)
- the importance of foreign trade for the *welfare* of individual countries and the world as a whole ($\rightarrow$ *gains from trade*)

Starting point is usually a self-reliant, closed economy ($\rightarrow$ autarky).
The extension refers to a transition to open economies ($\rightarrow$ international trade) and the consequences induced by it.

Real Foreign Trade Theory

A condition for international trade between two countries is in general – as will be proved below – a difference in national price ratios in autarky.
Reasons for autarky price differences

- different preferences in the countries
- different national abilities to produce certain goods ($\rightarrow$ technologies or factor endowments)
- different market conditions ($\rightarrow$ competitive conditions, government taxes and subsidies)

More recently, it has been proved that international trade can also be explained *without autarky price differences* by increasing returns to scale.

Real Foreign Trade Theory

Frequent assumptions for real trade theory

- perfect competition (compare Sec. 3 with Sec. 4)
- neutrality of money (exchange economy, theory of *relative* prices)
- general equilibria at full employment of all production factors
- international immobility of all factors of production at a perfectly price-inelastic factor supply
- no barriers to trade

Modern theory abolishes these assumption successively (e.g., introduction of barriers to trade in the form of tariffs or subsidies).

Monetary Trade Theory

Monetary trade theory is in essence a theory about equalizing the balance of payments (BOP). Characteristic problems:

- How do changes in certain economic values affect the BOP?
  - exchange rates ($\rightarrow$ exchange rate mechanism)
  - prices ($\rightarrow$ price mechanism, price-specie flow mechanism)
  - national income ($\rightarrow$ income mechanism)

  How do these mechanisms interact?
- How do changes in the BOP affect exchange rates, prices and income?
- Macroeconomics of open economies (internal and external equilibria)
- Determinants and effects of autonomous capital movements

Monetary theory goes beyond the scope of this course.

Empirical importance of international trade for Germany

Export data (goods and services) for Germany (at current prices)

figures in billions	1998 (DM)	2009 (Euro)	2017 (Euro)
GDP	3784.20	2397.10	3263.35
export	1092.12	978.75	1542.07
import	1028.85	860.31	1292.74
current account balance	63.27	118.48	249.33
export ratio$_{GDP}$	28.86 %	40.83 %	47.1 %
import ratio$_{GDP}$	27.19 %	35.89 %	39.6 %

Source: Jahresgutachten des SVR 1999/2000 and 2010/2011 and Eurostat

Those who need more detailed information about European trade read *External and intra-European Union trade, Statistical yearbook – Data 1958–2006, Eurostat Statistical Books, 2008, 471 pages.*

I Subjects and Empirical Importance
1.2 Empirical Facts on the International Division of Labor
Facts and Figures about Germany’s Foreign Trade

Export dependency of the German industry in 1998 measured by industial export ratios (foreign sales in relation to total sales):

intermediate goods	28.8 %
investment goods	47.9 %
consumer durables	26.8 %
consumer goods	17.2 %
total	33.1 %

Source: Institut der deutschen Wirtschaft Köln, figures on the economic development of Germany, 2000

Facts and Figures about Germany's Foreign Trade

Germany's most important trading partners

export	bn. Euro	import	bn. Euro
USA	106.90	China	93.82
France	101.38	NL	83.49
UK	86.15	France	65.69
NL	79.00	USA	57.90
China	76.09	Italy	51.82
Italy	61.44	Poland	46.46

Source: DeStatis, 2016

Export Ratios

Export ratios ($= Ex/GDP$) 2010 (international comparison)

USA	12.5 %	NL	77.9 %
Ger	46.2 %	Lux	186.7 %
UK	29.1 %	EU-16	38.9 %
Fra	25.1 %	EU-27	40.2 %

Source: Jahresgutachten des SVR 2009/2010; own calculations

$Y = C + I + Ex - Im$

Note for Luxemburg: $Ex > Y \implies C + I < Im$;
thus big parts of Im are re-exported as intermediate goods.

A First Explanation of Different Export Ratios

Export ratios apparently vary with the size and distance of a country.

Remark. The export ratios of the EU refer to all exports including those within the EU. Considering the EU as one unit, the calculation of exports to the rest of the world yields remarkably smaller export ratios, e.g., less than 14 % in 1998.

Remark. Import ratios are not mentioned here as they are very similar to export ratios ($\rightarrow$ international trade equilibrium or external equilibrium)

Biggest relative difference by far for NL and Lux;
Dutch import ratio 55.33 %, i.e. 5.5 percentage points less than the export ratio.

Economic Interlacing of the EU by International Trade

Intra-EU trade relations in 1995 (shares of exports to selected countries in total exports)

exports from \ to	Ger	UK	Fra	NL	EU-15
Ger	–	8.0	11.6	7.4	57.1
UK	12.9	–	9.8	7.9	57.1
Fra	17.7	9.3	–	4.6	63.5
NL	27.4	9.2	10.7	–	75.7
EU-15	13.9	7.7	10.1	5.8	63.2

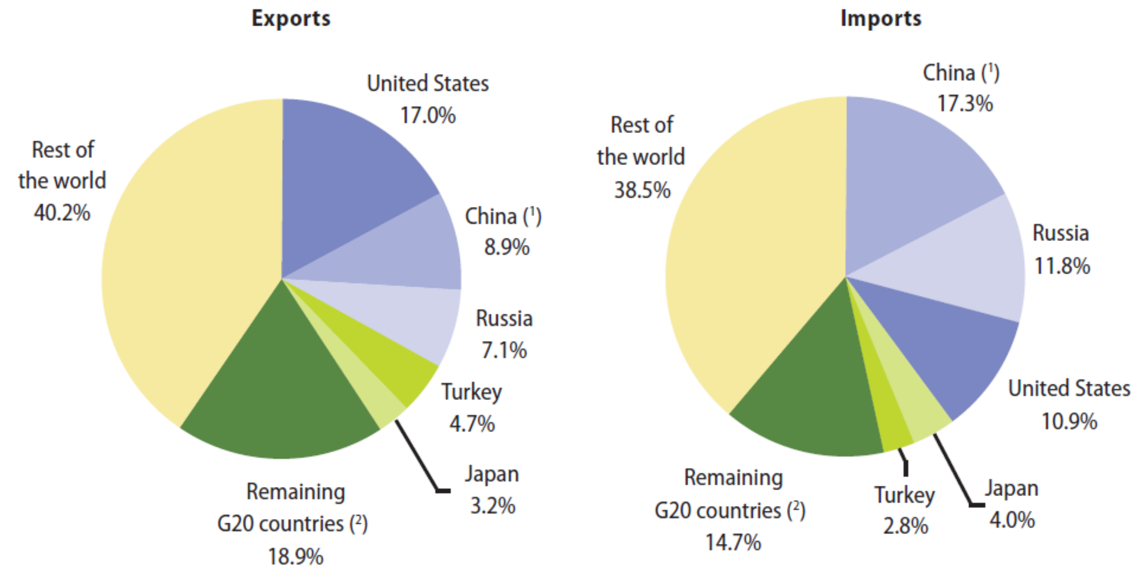
Source: W. Kortmann, Reale Außenwirtschaftslehre, Stuttgart 1998, p. 64.

The share of intra-regional trade for EU countries is extremely high. About two-thirds of the exports of all EU-15 countries go to neighboring EU-15 countries. (»We trade with our neighbors.«)

In the figure below the same holds true for the USA.

Economic Interlacing of the EU by International Trade

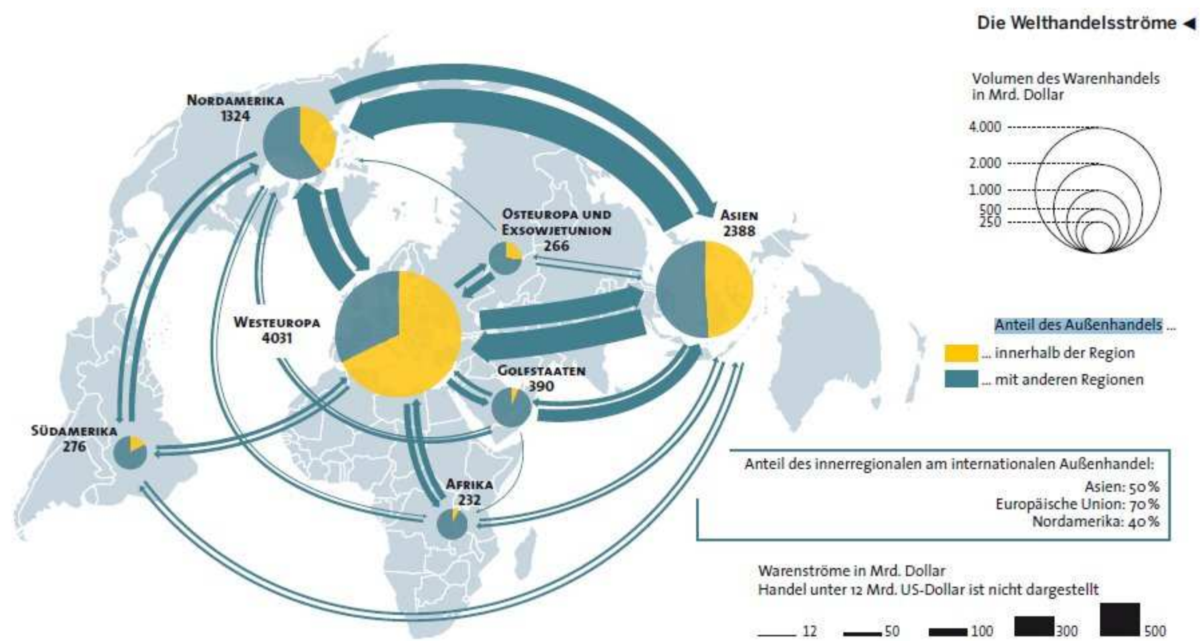
Extra-EU trade (EU-27, 2011) accounting for G20 main trading partners



(1) Excluding Hong Kong.
(2) Including Hong Kong.

Global Trade (international, intra-regional trade)

Global trade flows



Development of International Trade in the EU

The EU's foreign trade developped over time towards intra-regional trade ($\rightarrow$ intra-EU trade) and to the disadvantage of external trade ($\rightarrow$ extra-EU trade). The ratio between internal and external exports (similarly for imports) shows (Source: K. Heidensohn, Europe and World Trade, London 1995, pp. 10, 22; supplemented with data for 1995):

$$1958: \quad \frac{37\%}{63\%} \approx \frac{3}{5}, \qquad 1995: \quad \frac{63\%}{37\%} \approx \frac{5}{3}$$

(**European integration process.** 1958: EEC established; 1967: EC [EEC+ECSC+Euratom]; 1968: customs union; 1979: EMS & ERM; 1993: common market; 1993: EU; 1999: monetary union)

The figures overstate, however, because in the period at hand more an more countries acceded the EU (1972: UK, Irl, Den; 1985: Por, Spa, ...); so that external trade becomes internal trade by definition.

International Intra-Industry Trade

Importance of intra-industrial trade. Foreign trade between industrialized countries develops more and more towards *intra-industrial* trade, while international trade between industrialized and developing countries mainly remains to be *inter-industrial*.

Example. Trade of automobiles between Ger and Fra (or USA, Jap).

A key figure for the extent of intra-industrial trade is the

Grubel-Lloyd index: $$B_j = 1 - \frac{|Ex_j - Im_j|}{Ex_j + Im_j}$$

If the difference between exports and imports of some product group j is big in relation to the sum of exports and imports, intra-industrial trade is of minor importance.

In extreme no intra-industrial trade takes place if either $Ex_j = 0$ or $Im_j = 0$ so that $B_j = 0$.

The maximum intra-industrial trade occurs if $Ex_j = Im_j$ with $B_j = 1$.

International Intra-Industry Trade

Hints.

- B_j increases with the aggregation level. If we focus only on passenger cars VW Golf, intra-industrial trade cannot take place. This changes if we treat the class of compact cars.
- Grubel-Lloyd indices for product groups, e.g. all traded goods or manufactures, are weighted averages of less aggregated groups.
- The figures in the following table are based on the UN's *Standard International Trade Classification (SITC Rev. 4)* and, especially, on data of the *3-digit-level*. Example: top level SITC class 7: machinery and transport equipment. The 3-digit-class 752 embraces automatic data-processing machines.

FYI: SITC Rev. 4

UN's *Standard International Trade Classification* (SITC Rev. 4)	
0	Food and live animals
1	Beverages and tobacco
2	Crude materials, inedible, except fuels
3	Mineral fuels, lubricants and related materials
4	Animal and vegetable oils, fats and waxes
5	Chemicals and related products
6	Manufactured goods (classified chiefly by material)
7	Machinery and transport equipment
75	Office machines and automatic data-processing machines
78	Road vehicles
781	*Motor cars and other motor vehicles principally designed for the transport of persons*
8	Miscellaneous manufactured articles
9	Commodities and transactions not classified elsewhere in the SITC

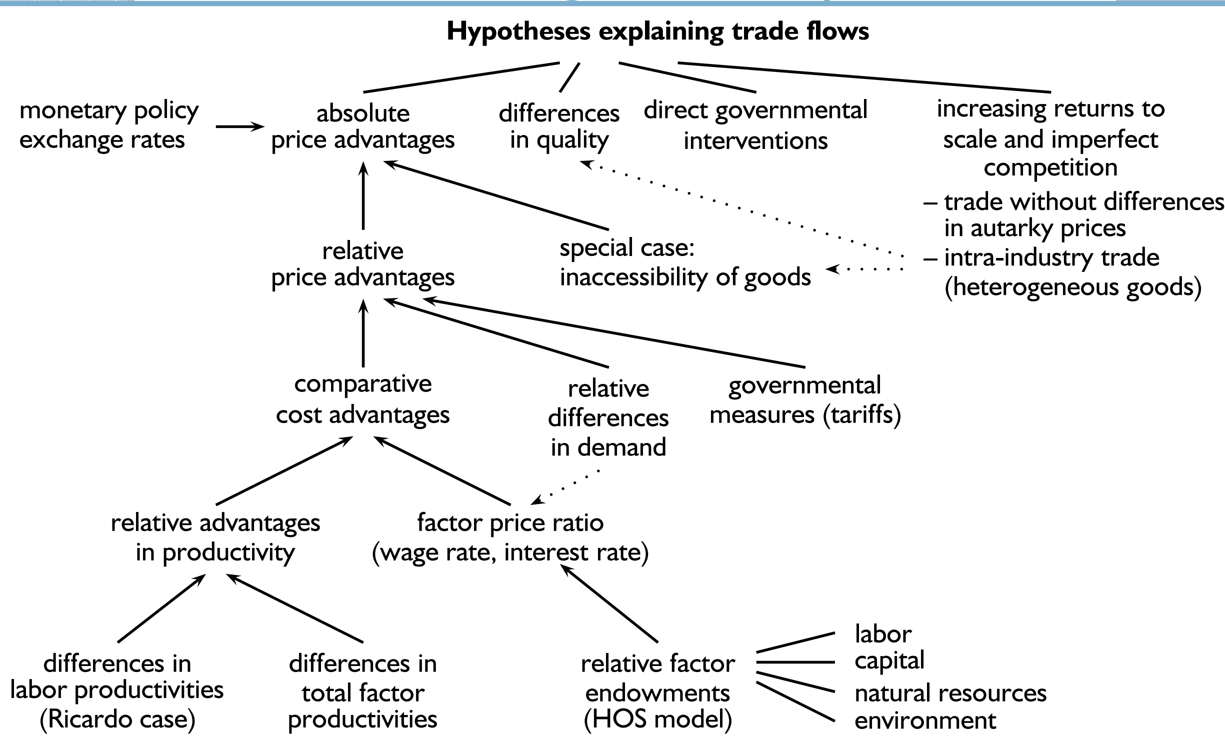
Grubel-Lloyd Index

Grubel-Lloyd index on the basis of three digit levels

	all traded goods		industrial outputs (manufactures)	
	1970	1985	1970	1985
Ger	0.54	0.63	0.60	0.67
Fra	0.67	0.74	0.78	0.82
USA	0.53	0.54	0.57	0.61
Jap	0.26	0.23	0.32	0.26

Source: K. Heidensohn, Europe and World Trade, London 1995, p. 26.

Overview on Real Foreign Trade Theory



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Assumptions

Assumptions for the production sector

- perfect competition with numerous small firms
- industry specific production functions all being homogeneous of degree one ($\rightarrow$ constant returns to scale)
- no barriers to market entrance and all firms of an industry (potentially) have available the same production technology
- factors of production are supplied completely price inelastic at given amounts

Purpose. Describe competitive equilibria in input and output markets.

Remark. In this course we derive several theorems. In many cases we provide arguments of plausibility rather than strict proofs. This is not mentioned everywhere.

Revenue Maximization vs. Profit Maximization

Statement. If all *individual* firms maximize their profits then they behave in *common* as if they would maximize the total revenue (real national income).

A proof follows from the comparison of optimum conditions

- of national revenue maximization and
- of individual profit maximization
  (determinants for the behavior of firms as price takers)

If this is correct, it suffices to represent the supply side of an economy by solving the national revenue maximization problem.

Revenue Maximization

The production possibility set is the technical constraint for revenue maximization (see below).

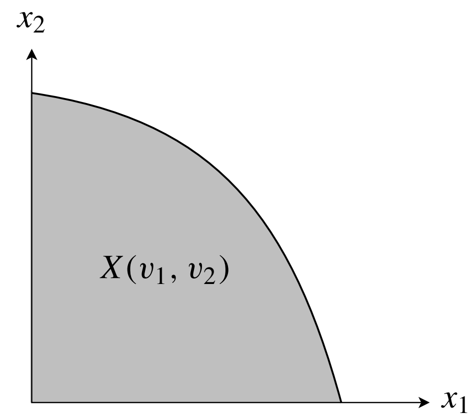
Symbols. $p_1, p_2 =$ given prices of goods; $x_1, x_2 =$ quantities of goods; $v_1, v_2 =$ given quantities of production factors (factor endowment); $f_1, f_2 =$ production functions of industries; $v_{ij} =$ quantity of input i in the production of output j.

The production possibility set denotes all output combinations which are technically feasible at given production techniques and given quantities of resources.

$$X(v_1, v_2) := \Big\{(x_1, x_2) \geqq (0,0) \mid x_j \leq f_j(v_{1j}, v_{2j}), \\ (v_{1j}, v_{2j}) \geqq (0,0),\ j = 1, 2; \\ v_{i1} + v_{i2} \leq v_i,\ i = 1, 2\Big\}$$

Revenue Maximization

Graphical representation of the production possibility set or production block $X(v_1, v_2)$. Its north-east frontier is called the transformation curve or production frontier which denotes the maximum output of good 2 at a fixed quantity of good 1 given the production techniques and given factor endowments. The curve is concave under the assumptions made before (increasing or constant marginal rate of transformation $MRT = -\mathrm{d}x_2/\mathrm{d}x_1$).



x_1 = manufactures,
x_2 = services,
v_1 = national capital stock,
v_2 = national labor force

Revenue Maximization

Revenue function

$$\tilde{r}(p_1, p_2, v_1, v_2) = \max_{x_1, x_2} \{p_1 x_1 + p_2 x_2 \mid (x_1, x_2) \in X(v_1, v_2)\}$$

Optimal quantities (→ supply) depend on the parameters p_1, p_2, v_1, and v_2.

$$\hat{x}_1 = \tilde{x}_1^S(p_1, p_2, v_1, v_2), \quad \hat{x}_2 = \tilde{x}_2^S(p_1, p_2, v_1, v_2)$$

Substitution into the objective function

$$\tilde{r}(p_1, p_2, v_1, v_2) = p_1 \tilde{x}_1^S(p_1, p_2, v_1, v_2) + p_2 \tilde{x}_2^S(p_1, p_2, v_1, v_2)$$

The revenue function denotes the maximum production value (or real national income, GNI) feasible at given prices and given quantities of production factors.

Revenue Maximization

Necessary conditions for a revenue maximum are derived here under simplifying assumptions (no inequalities).
Lagrangean function

$$L = p_1 f_1(v_{11}, v_{21}) + p_2 f_2(v_{12}, v_{22}) + \lambda_1(v_1 - v_{11} - v_{12}) + \lambda_2(v_2 - v_{21} - v_{22})$$

Necessary first order conditions for a revenue maximum

$$(a) \quad \frac{\partial L}{\partial v_{11}} = p_1 \frac{\partial f_1}{\partial v_{11}} - \lambda_1 = 0 \qquad (b) \quad \frac{\partial L}{\partial v_{21}} = p_1 \frac{\partial f_1}{\partial v_{21}} - \lambda_2 = 0$$

$$(c) \quad \frac{\partial L}{\partial v_{12}} = p_2 \frac{\partial f_2}{\partial v_{12}} - \lambda_1 = 0 \qquad (d) \quad \frac{\partial L}{\partial v_{22}} = p_2 \frac{\partial f_2}{\partial v_{22}} - \lambda_2 = 0$$

$$(e) \quad \frac{\partial L}{\partial \lambda_1} = v_1 - v_{11} - v_{12} = 0 \qquad (f) \quad \frac{\partial L}{\partial \lambda_2} = v_2 - v_{21} - v_{22} = 0$$

2 General Equilibrium in the Closed Economy (Autarky)
2.1 Competitive Equilibrium in the Production Sector
Revenue Maximization

(a)—(c) and (b)—(d) imply

$$p_1 \frac{\partial f_1}{\partial v_{11}} = p_2 \frac{\partial f_2}{\partial v_{12}} = \lambda_1 \quad \text{and} \quad p_1 \frac{\partial f_1}{\partial v_{21}} = p_2 \frac{\partial f_2}{\partial v_{22}} = \lambda_2,$$

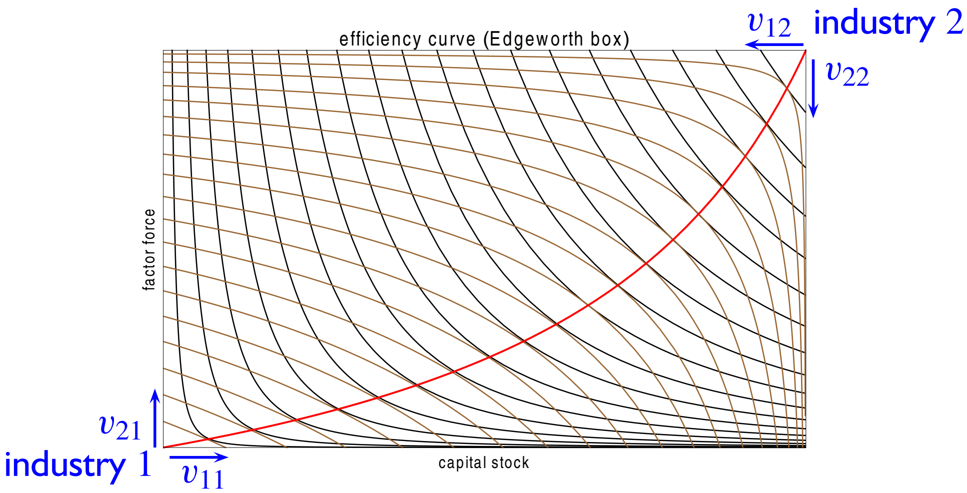
that is the equalization of monetary marginal productivities.
The division of both equations yields

$$(1) \qquad \frac{\partial f_1/\partial v_{11}}{\partial f_1/\partial v_{21}} = \frac{\partial f_2/\partial v_{12}}{\partial f_2/\partial v_{22}} = \frac{\lambda_1}{\lambda_2} \iff -\frac{\mathrm{d}v_{21}}{\mathrm{d}v_{11}} = -\frac{\mathrm{d}v_{22}}{\mathrm{d}v_{12}}$$

that is the marginal rates of (technical) substitution (MRS) correspond in both industries. This condition guarantees an efficient factor allocation at full employment (conditions (e) and (f)). Each factor allocation corresponds to a distinct output combination on the transformation curve.

Revenue Maximization

Competitive equilibria in input markets. The Edgeworth box represents full employment of both factors of production, their assignment to industries (factor allocation), and matching marginal rates of substitution (efficiency curve).



Revenue Maximization

(a) to (d) imply also

$$\frac{p_1}{p_2} = \frac{\partial f_2/\partial v_{12}}{\partial f_1/\partial v_{11}} = \frac{\partial f_2/\partial v_{22}}{\partial f_1/\partial v_{21}},$$

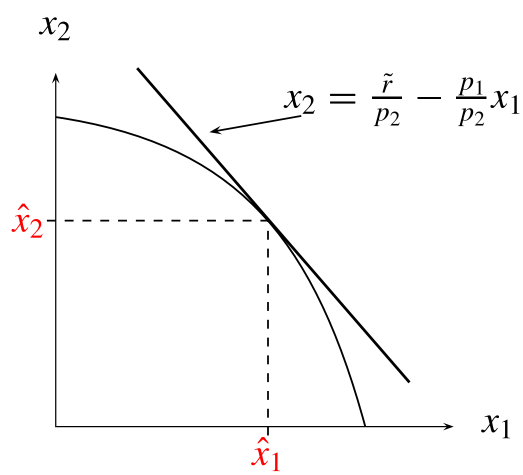
where one can show that the expression on the right hand side corresponds to the marginal rate of transformation (MRT), $-\mathrm{d}x_2/\mathrm{d}x_1$. Hence

(2) $$\frac{p_1}{p_2} = -\frac{\mathrm{d}x_2}{\mathrm{d}x_1} = MRT$$

The MRT equals the inverse price ratio at the optimum. If the transformation curve is strictly concave, the optimum solution for a revenue maximum is unique (tangent solution).

Revenue Maximization

Competitive equilibrium in output markets.
Revenue maximization means to shift the line $\tilde{r} = p_1 x_1 + p_2 x_2$ outwards as far as possible so that the depicted tangent solution results. The bundle $(\hat{x}_1, \hat{x}_2)$ is called the production point. The slope of the revenue line is the negative commodity price ratio $-p_1/p_2$. Therefore, the conditions (2) and implicitly (1) hold good in the optimum.



Profit maximization

The profit of industry 1 is

$$\pi_1 = p_1 f_1(v_{11}, v_{21}) - q_1 v_{11} - q_2 v_{21}.$$

Necessary first order conditions for a profit maximum

$$\text{(a')} \quad p_1 \frac{\partial f_1}{\partial v_{11}} - q_1 = 0, \qquad \text{(b')} \quad p_1 \frac{\partial f_1}{\partial v_{21}} - q_2 = 0.$$

Similarly for the second industry

$$\text{(c')} \quad p_2 \frac{\partial f_2}{\partial v_{12}} - q_1 = 0, \qquad \text{(d')} \quad p_2 \frac{\partial f_2}{\partial v_{22}} - q_2 = 0.$$

Remark. The factors are paid according to their monetary marginal productivities. Both industries show identical factor prices.

The four conditions imply (1) because

$$\frac{q_1}{q_2} = \frac{\partial f_1/\partial v_{11}}{\partial f_1/\partial v_{21}} = \frac{\partial f_2/\partial v_{12}}{\partial f_2/\partial v_{22}} \qquad \text{(MRS)}.$$

Similarly, (2) can be computed

$$\frac{p_1}{p_2} = \frac{\partial f_2/\partial v_{12}}{\partial f_1/\partial v_{11}} = \frac{\partial f_2/\partial v_{22}}{\partial f_1/\partial v_{21}} = -\frac{\mathrm{d}x_2}{\mathrm{d}x_1} \qquad \text{(MRT)}.$$

Eventually, if

$$\hat{\lambda}_1 = q_1 \quad \text{and} \quad \hat{\lambda}_2 = q_2,$$

the firms chose the same input quantities (and consequently output quantities) as in the revenue maximization problem.

(We do not prove explicitely here that the Lagrangean multipliers correspond to factor prices.)

As a consequence the conditions (e) and (f) are realized as equilibrium conditions for the factor markets.

$$\underbrace{v_1}_{\text{supply}} = \underbrace{v_{11} + v_{12}}_{\text{demand}}, \qquad \underbrace{v_2}_{\text{supply}} = \underbrace{v_{21} + v_{22}}_{\text{demand}}$$

Supply Side Represented by Revenue Maximization

Conclusion. The equilibrium conditions under perfect competition correspond exactly to the optimum conditions of revenue maximization. Thus, the production sector can be represented by the revenue function, which is easier to handle. The revenue maximizing outputs are the quantities supplied under perfect competition.

$$\hat{x}_1 = \tilde{x}_1^S(p_1, p_2, v_1, v_2), \ \ \hat{x}_2 = \tilde{x}_2^S(p_1, p_2, v_1, v_2)$$

Note again that the revenue corresponds to the real national income for the present case. That is we maximize the real income in each country.

The Meaning of Relative Prices

The optimum quantities $\hat{x}_1$ and $\hat{x}_2$ do not change if all prices are multiplied by the same constant $\mu > 0$. (Indicated by the graphical solution as $\mu p_1/\mu p_2 = p_1/p_2 =: p$.) The supply functions are, therefore, homogeneous of degree 0 in their prices and (dropping the factor endowments) they can be rewritten as (choose $\mu = 1/p_2$):

$$\mu^0 \tilde{x}_1^S(p_1, p_2) = \tilde{x}_1^S(\mu p_1, \mu p_2) = \tilde{x}_1^S(p_1/p_2, 1) =: x_1^S(p),$$
$$\mu^0 \tilde{x}_2^S(p_1, p_2) = \tilde{x}_2^S(\mu p_1, \mu p_2) = \tilde{x}_2^S(p_1/p_2, 1) =: x_2^S(p)$$

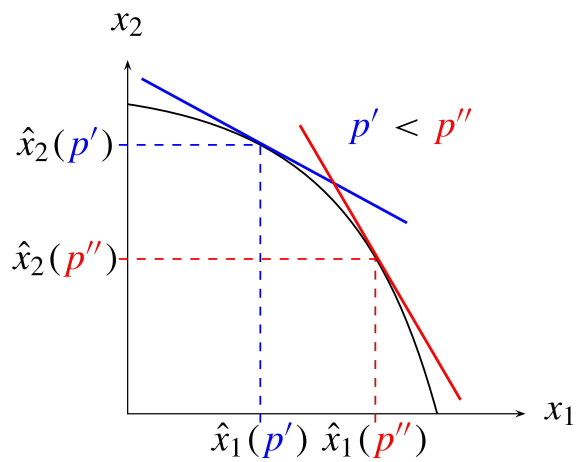
The revenue (homogeneous of degree 1 in prices) can also be expressed as a function of p (hence in units of good 2)

$$\mu \tilde{r}(p_1, p_2) = \tilde{r}(\mu p_1, \mu p_2) = \tilde{r}(p, 1) =: r(p)$$
$$\text{or} \quad r(p) = p x_1^S(p) + x_2^S(p)$$

Dependence of the Supply of Goods on Relative Prices

The graphical representation of the revenue maximization problem indicates the dependence on prices:

$$\frac{\mathrm{d}x_1^S(p)}{\mathrm{d}p} > 0, \quad \frac{\mathrm{d}x_2^S(p)}{\mathrm{d}p} < 0$$



Summary. Given factor endowments v_1, v_2 and given production techniqes, the relative price p determines the production point.

Dependence of the Supply of Goods on Relative Prices

Remark. The quantities supplied depend also on the factor endowments.

$$x_1^S(p, v_1, v_2) \quad \text{and} \quad x_2^S(p, v_1, v_2)$$

This will be discussed with regard to the Rybczinski theorem at a later stage.

End of production sector

Utility Maximization

Consumption

The basic decision problem of a household is to determine a bundle of goods (x_1, x_2) such that the given utility function $u(x_1, x_2)$ attains the maximum value having regard to a budget constraint $p_1x_1 + p_2x_2 \leq \tilde{y}$ with given commodity prices (p_1, p_2) and a given income $\tilde{y}$.

$$\max_{x_1, x_2} \{u(x_1, x_2) | \; p_1x_1 + p_2x_2 \leq \tilde{y}\}$$

The function values of the optimum bundle of commodities $(\hat{x}_1, \hat{x}_2)$ depend on (p_1, p_2) and $\tilde{y}$. A variation of these parameters generates the Marshallian demand functions

$$\hat{x}_1 = \tilde{x}_1^M(p_1, p_2, \tilde{y}) \quad \text{and} \quad \hat{x}_2 = \tilde{x}_2^M(p_1, p_2, \tilde{y}).$$

Example: Cobb-Douglas function

Assume the utility function $u(x_1, x_2) = ax_1^\alpha x_2^\beta$ and the budget constraint $p_1x_1 + p_2x_2 = \tilde{y}$. Lagrangean function:

$$L(x_1, x_2, \lambda) = u(x_1, x_2) + \lambda(\tilde{y} - p_1x_1 - p_2x_2)$$

First order necessary conditions for a utility maximum:

$$\begin{aligned}
\frac{\partial L}{\partial x_1} &= \frac{\partial u}{\partial x_1} - \lambda p_1 = a\alpha x_1^{\alpha-1}x_2^\beta - \lambda p_1 \stackrel{!}{=} 0 \\
\frac{\partial L}{\partial x_2} &= \frac{\partial u}{\partial x_2} - \lambda p_2 = a\beta x_1^{\alpha}x_2^{\beta-1} - \lambda p_2 \stackrel{!}{=} 0 \\
\frac{\partial L}{\partial \lambda} &= \tilde{y} - p_1x_1 - p_2x_2 \stackrel{!}{=} 0
\end{aligned}$$

Using these three equations for the determination of the three variables x_1, x_2 and λ we gain the Marshallian demand functions:

$$\tilde{x}_1^M(p_1, p_2, \tilde{y}) = \frac{\alpha}{\alpha+\beta}\frac{\tilde{y}}{p_1}, \quad \tilde{x}_2^M(p_1, p_2, \tilde{y}) = \frac{\beta}{\alpha+\beta}\frac{\tilde{y}}{p_2}$$

Example: Cobb-Douglas function

Marshallian demand functions are homogeneous of degree 0 in prices *and* income ($\rightarrow$ freedom of monetary illusion).

$$\mu^0 \hat{x}_j = \hat{x}_j = \tilde{x}_j^M(\mu p_1, \mu p_2, \mu \tilde{y}), \quad j = 1, 2$$

They can, therefore, be rewritten as functions of the relative price $p := p_1/p_2$ and the real income $y := \tilde{y}/p_2$ measured in units of good 2 (choose again $\mu = 1/p_2$).

In the Cobb-Douglas case we find

$$x_1^D(p, y) := \tilde{x}_1^M(p_1/p_2, 1, \tilde{y}/p_2) = \frac{\alpha}{\alpha+\beta}\frac{y}{p}$$

$$x_2^D(p, y) := \tilde{x}_2^M(p_1/p_2, 1, \tilde{y}/p_2) = \frac{\beta}{\alpha+\beta} y$$

Aggregate Demand

Aggregate demand functions unfortunately have almost always different properties compared to individual demand functions. Moreover, capturing all individual utility functions would burst our theoretical limits ($\rightarrow$ notably the income distribution).

Possible solutions

- Assume that all consumers have identical homothetic preferences $\rightarrow$ the aggregation problem becomes solvable.
- Everybody consumes a positive amount of each good, then the aggregation of utility function holding the Gorman form is possible (allows differences in individual preferences).
- An optimum income distribution may enable us to use Samuelsons' social welfare function.

In these cases one can show that the aggregate demand has the same properties as the behavior of a representative consumer.

Homothetic Functions

Disregarding the theoretical background of homothetic preferences we use them with the following unproven advantages.

- The analysis of a representative consumer suffices.
- The height of his income and the number of consumers do not influence the consumption ratio x_2/x_1.
- The aggregate demand is independent of the distribution of incomes and it has all the properties of an individual demand function ($\rightarrow$ Slutzky constraint).
- The *relative price* $p = p_1/p_2$ determines the consumption point.

2 General Equilibrium in the Closed Economy (Autarky)
2.2 The Consumption Sector — Demand and Welfare

Homothetic Functions

On the assumption of identical homothetic utility functions

The class of *homothetic utility functions* is formed by all utility function that are homogeneous of degree one and their increasing monotonic transformations.

Definition. A function U is homothetic if it can be written in the form $U = F\left[f(x_1, x_2)\right]$ where F is a continuous, strictly monotonically increasing function of one variable with $F(0) = 0$ and where f is a linearly homogeneous function of (x_1, x_2).

Useful properties (cf. the figure below):

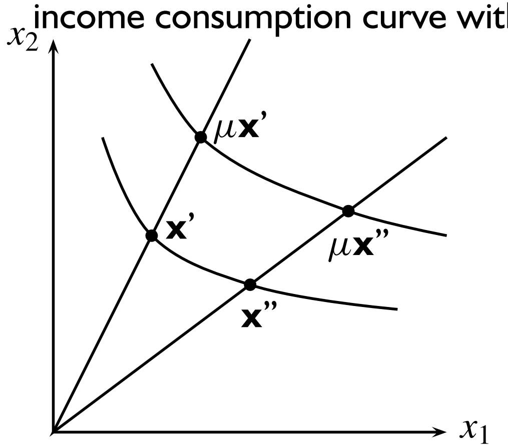
(a) Indifferent bundels of goods remain indifferent after a scaling.

(b) The MRS is constant along rays through the origin.

(c) The income consumption curves are rays through the origin.

Homothetic Functions

income consumption curve with $x_1/x_2 = \text{const.}$



Ad (a) Indifferent bundels of goods remain indifferent after scaling.

$$u(x_1', x_2') = u(x_1'', x_2'') \implies u(\mu x_1', \mu x_2') = u(\mu x_1'', \mu x_2'')$$

Homothetic Functions

Ad (b) The MRS is constant along rays through the origin.
Example for a Cobb-Douglas utility function

$$-\frac{\mathrm{d}x_2}{\mathrm{d}x_1} = \frac{\partial u/\partial x_1}{\partial u/\partial x_2} = \frac{p_1}{p_2} = \frac{\alpha}{\beta}\frac{x_2}{x_1} = \text{const.}$$

Ad (c) The income consumption curves are rays through the origin or variations of income do not change the ratio x_2^D/x_1^D
Example for a Cobb-Douglas utility function

$$\frac{x_2^M(p_1, p_2, \tilde{y})}{x_1^M(p_1, p_2, \tilde{y})} = \frac{x_2^D(p, y)}{x_1^D(p, y)} = \frac{\beta}{\alpha}\frac{\tilde{y}/p_2}{\tilde{y}/p_1} = \frac{\beta}{\alpha}\frac{p_1}{p_2} = \frac{\beta}{\alpha}p = \text{const.}$$

Homothetic Functions

Summary for identical homothetic utility functions

- The amount of income and the number of consumers have no impact on the consumption ratio x_2/x_1.
- Aggregate demand is independent of any income distribution and it has all properties of an individual demand function (→ Slutzky constraint).
- It suffices to analyze a representative houshold.
- The relative price determines the consumption point.

Remark. All homogeneous functions are homothetic but not vice versa. Homogenity is a cardinal property; this assumption does not fit to approaches based on ordinal utility functions.

Homothetic Functions

Assuming that individual preferences can be aggregated so that community indifference curves exist we can use them as follows.

(a) Positive interpretation. Given prices and aggregate income we determine the quantities demanded by the highest community indifference curve tangent to the budget constraint.

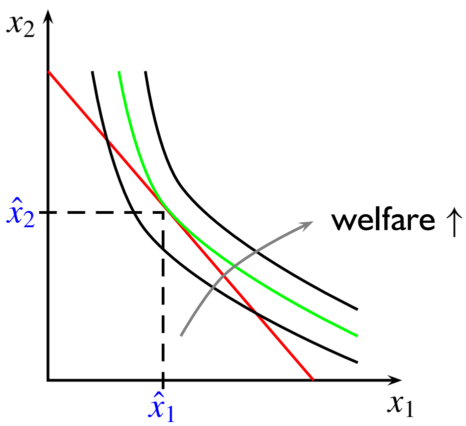
(b) Normative interpretation. Moving from lower to higher community indifference curves indicates a positive welfare significance, although such a movement does not mean that the welfare of all individuals has increased.

The importance of the last objection will be explained with regard to the Stolper-Samuelson theorem.

Community Welfare (Graphical Representation)

Maximizing the community's welfare refers to a community welfare function of aggregate individual preferences. Commodity prices p_1 and p_2 as well as the real national income $\tilde{y}$ are given. (Cf. p. 45.)

$$\max_{x_1, x_2} \{u(x_1, x_2) | \; p_1 x_1 + p_2 x_2 \leq \tilde{y}, \; x_1 \geq 0, x_2 \geq 0\}$$



optimum condition:

marginal rate of substitution (MRS)

$$-\frac{dx_2}{dx_1} = \frac{\frac{\partial u(\hat{x}_1, \hat{x}_2)}{\partial x_1}}{\frac{\partial u(\hat{x}_1, \hat{x}_2)}{\partial x_2}} = \frac{p_1}{p_2}$$

Solution $(\hat{x}_1, \hat{x}_2)$ = consumption point (aggregate demand)

Supply and Demand in a General Equilibrium Model

General equilibrium

In a general equilibrium model the production cost and the residual profits are equal to the incomes of the households. The total income is entirely consumed (no saving) and it corresponds to the total revenue of the firms.

$$y = r(p)$$

The aggregate demand functions (p. 47) are

$$x_1^D(p, r(p)) \quad \text{and} \quad x_2^D(p, r(p)).$$

Any equilibrium requires a match of supply (p. 42) and demand.

$$x_1^S(p) = x_1^D(p, r(p))$$
$$x_2^S(p) = x_2^D(p, r(p))$$

Supply and Demand in a General Equilibrium Model

With this we have two equations to determine only one variable $(p = p_1/p_2)$.

One might think that the system of equations would be overdetermined and the absolute prices could be computed. Walras' law, however, states that both equation depend on each other so that just one independent equation remains.

A real model (without money) suffices in general to compute only relative prices.

If we use $p = p_1/p_2$ [unit$_2$/unit$_1$], commodity 2 is called the numéraire (counting unit). Adding money as additional good it becomes usually the numéraire (cf. Dixit/Norman, Ch. 7).

Walras' Law

A proof of Walras' law regarding the case of two goods follows immediately from the household's budget constraint noted with absolute prices.

$$p_1\tilde{x}_1^M + p_2\tilde{x}_2^M = \tilde{y} = \tilde{r} = p_1\tilde{x}_1^S + p_2\tilde{x}_2^S.$$

A simple restatement yields

$$p_1(\tilde{x}_1^M - \tilde{x}_1^S) + p_2(\tilde{x}_2^M - \tilde{x}_2^S) = 0$$

and hence

$$\tilde{x}_1^M = \tilde{x}_1^S \implies \tilde{x}_2^M = \tilde{x}_2^S$$

More generally, this is Walras' law.

If $n-1$ out of n markets are equilibrated then the nth market must also show an equilibrium.

Properties of an Equilibrium

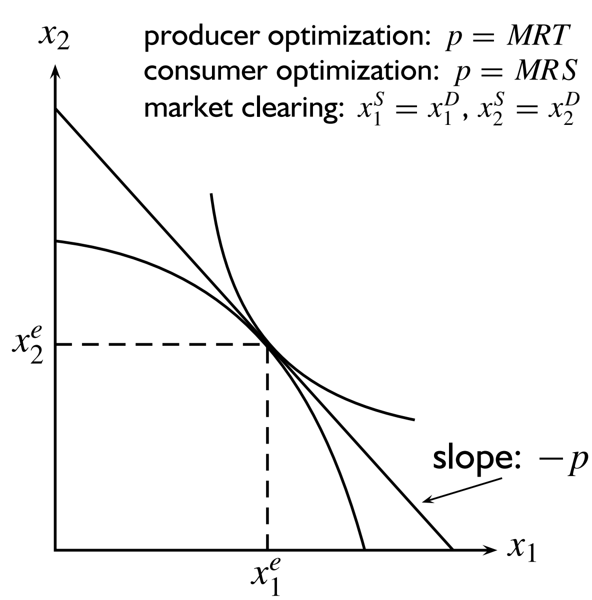
Basic questions with regard to equilibria.

- existence: ensured in general
- uniqueness: ensured under the assumptions made (strictly convex indifference curves, convex production possibility set)
- stability: ensured for a Walrasian process of price formation under the assumptions made

Uniqueness and stability are not always guaranteed in more general settings.

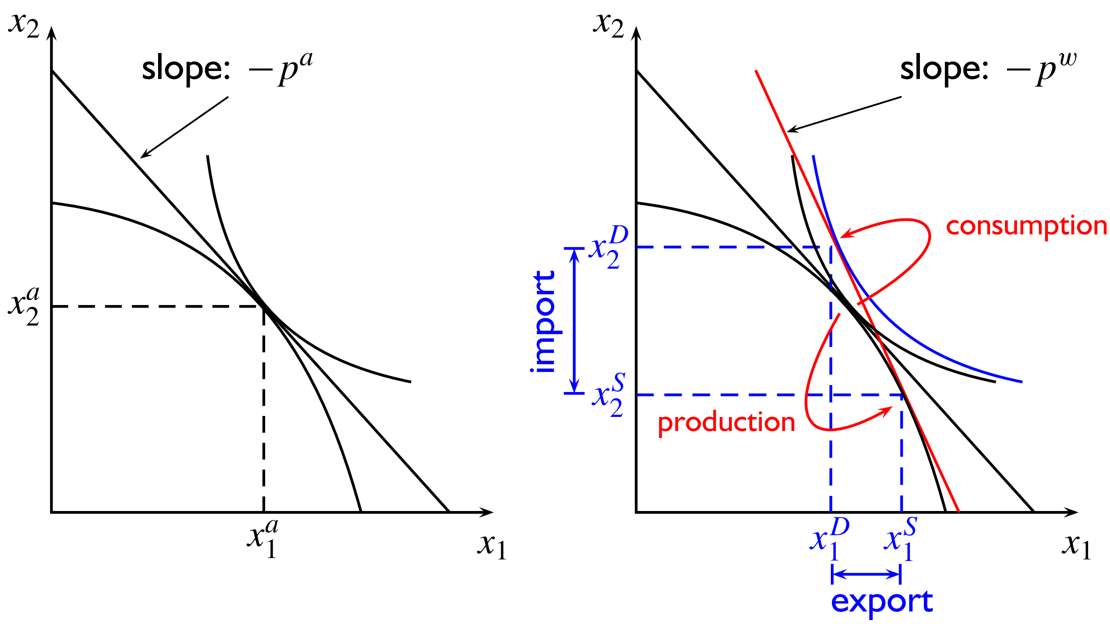
Graphical Presentation of an Equilibrium

A competitive general equilibrium in autarky results from the tangent point of the indifference curve and the transformation curve. The common slope at this point determines the equilibrium price ratio p. The line corresponds to the revenue (national income) from the industries' point of view and to the budget from the consumers' point of view.



Graphical Presentation of an Equilibrium

Transition from autarky to international trade



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General Equilibrium for Two Countries

Two countries. Distinguish the home country from the rest of the world indicated by the superscript $*$.

Market clearing for both world markets

$$x_j^D(p, r(p)) + x_j^{D*}(p, r^*(p)) = x_j^S(p) + x_j^{S*}(p), \quad j = 1, 2.$$

Remark. One variable (the relative price $p = p_1/p_2 = p_1^*/p_2^*$), but two equations.
As before one equation is redundant (Walras' law, cf. p. 68) so that there is only one independent equation.

Hint. The exchange rate e denoting the price of a foreign currency does not exist in a real model. Independent of its hypothetical value the exchange rate does not affect relative prices. For example using good 2 as numéraire the prices $p_1 = ep_1^*$ and $p_2 = ep_2^*$ yield $p_1/p_2 = p_1^*/p_2^*$ independent of e .

Excursion. Price and Quantity Notation

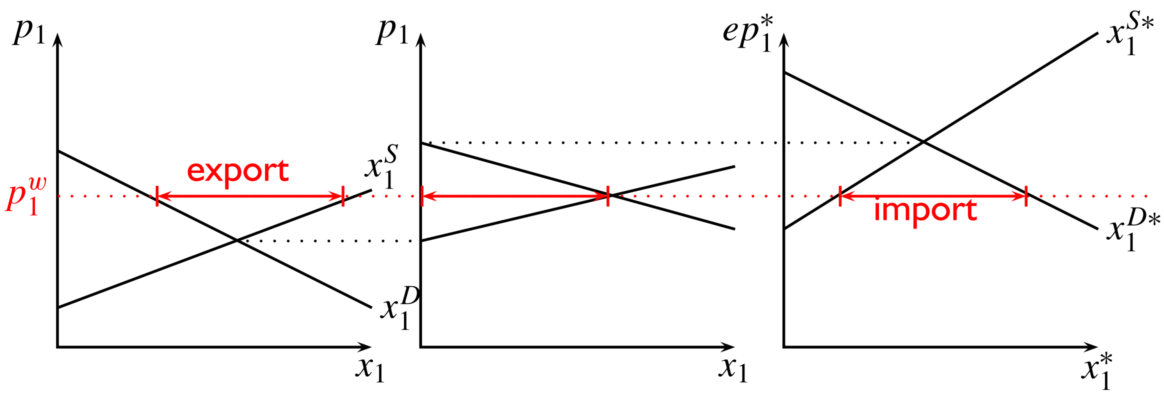
The exchange rate regarding, e.g., euro and dollar can be denoted in two ways. We always (!) use the price notation. The rate of exchange is the price of a foreign currency unit denoted in euro e [€/$].

If the exchange rate denotes the price of a domestic currency unit in units of a foreign currency, i.e. $1/e$ [$/€], it is called the quantity notation.

Excursion. World Export Market

World export market (partial equilibrium! Ignored backlashes on p_2 and y)

export condition regarding autarky prices $p_1 < ep_1^*$
equilibrium world market price $p_1 < p_1^w < ep_1^*$ with export = import



3 Competitive General Equilibrium in Open Economies
3.1 General Equilibrium Allowing for Foreign Trade

Excess Demand Function

Definition of excess demand (for both goods $j = 1, 2$):

$$m_j(p) := x_j^D(p, r(p)) - x_j^S(p), \quad m_j^*(p) := x_j^{D*}(p, r^*(p)) - x_j^{S*}(p).$$

This means for the home county
$m_j < 0$: export of good j
$m_j > 0$: import of good j

The equilibrium conditions can now be reformulated as

$$m_j = x_j^D - x_j^S = -(x_j^{D*} - x_j^{S*}) = -m_j^*,$$

or

$$m_j(p) + m_j^*(p) = 0, \quad j = 1, 2.$$

A domestic excess demand must be matched by a foreign excess supply et vice versa.

Balance of Payments Equilibrium

Example. The home country exports good 1 and imports good 2 (this direction of trade is assumed throughout the entire course). Using the country's budget constraint ($\tilde{y} = \tilde{r}$) with *money prices* we have

$$p_1(\tilde{x}_1^M - \tilde{x}_1^S) + p_2(\tilde{x}_2^M - \tilde{x}_2^S) = -Ex^€ + Im^€ = 0.$$

The current account balance Z turns out to be (reversed signs)

$$Z = Ex^€ - Im^€ = 0.$$

Similarly, we find $Z^* = 0$ and $Z^* \equiv -Z$. ($\rightarrow$ balanced trade).

Same statement in the *relative price* p

$$p(x_1^D - x_1^S) + (x_2^D - x_2^S) = pm_1 + m_2 = 0$$

International General Equilibrium

Using $Z + Z^* = 0$ we have (remember $p = p_1/p_2 = p_1^*/p_2^*$)

$$p\left(m_1(p) + m_1^*(p)\right) + \left(m_2(p) + m_2^*(p)\right) = 0.$$

According to Walras' law it suffices to consider just one equilibrium condition (m_1^w = world excess demand function for good 1).

$$m_1^w(p) := m_1(p) + m_1^*(p) = 0$$

The analysis below is focused on properties of an international general equilibrium (more details follow)

- existence: find some p_e so that $m_1^w(p_e) = 0$
- stability: prove that $m_1^w(p)$ decreases in p at the equilibrium
- uniqueness: prove that all equilibria are stable

Oniki-Uzawa Offer Curves

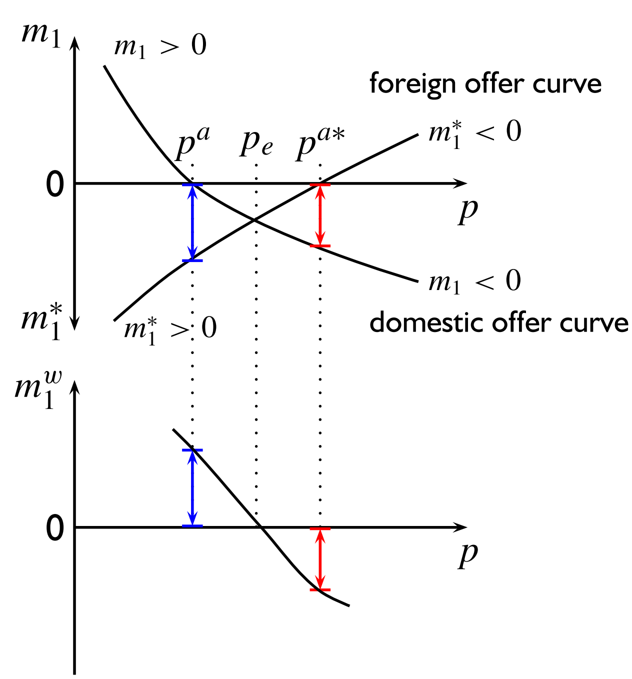
- A graphical illustration of the international general equilibrium uses Oniki-Uzawa offer curves to represent the excess demand functions $m_1(p)$ and $m_1^*(p)$.
- The analytical examination of $m_1(p)$, $m_1^*(p)$, and $m_1^W(p)$ requires knowledge about duality theory. We need particularly the Slutzky equation and the envelope theorem.

Oniki-Uzawa Offer Curves

World excess demand

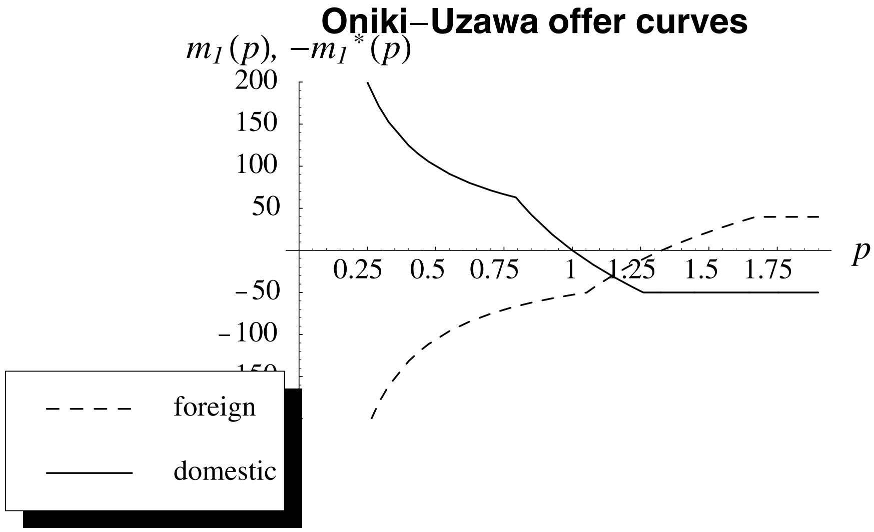
$$m_1^w := m_1 + m_1^*$$

At the price p_e we find $m_1 + m_1^* = 0$, i.e. p_e is the equilibrium price ratio at the world market. The home country exports good I ($m_1 < 0$; excess supply). The relative world market price *must* lie between the relative autarky pricess (p^a and p^{a*}).



Oniki-Uzawa Offer Curves

Example from WISU paper (linearly homogeneous utility and production functions (all of Cobb-Douglas type) for both countries)



Oniki-Uzawa Offer Curves

Slope of the domestic excess demand function regrading good 1 (assume strictly convex indifference curves and a strictly concave transformation curve)

$$\begin{aligned}
\frac{\mathrm{d}m_1(p)}{\mathrm{d}p} &= \frac{\mathrm{d}x_1^M(p, r(p))}{\mathrm{d}p} - \frac{\mathrm{d}x_1^S(p)}{\mathrm{d}p} \\
&= \underbrace{\frac{\partial x_1^M}{\partial p}}_{\text{Slutzky}} + \frac{\partial x_1^M}{\partial r}\underbrace{\frac{\mathrm{d}r}{\mathrm{d}p}}_{=x_1^S} - \frac{\mathrm{d}x_1^S}{\mathrm{d}p} \\
&= \frac{\partial x_1^H}{\partial p} - \frac{\partial x_1^M}{\partial r}x_1^M + \frac{\partial x_1^M}{\partial r}x_1^S - \frac{\mathrm{d}x_1^S}{\mathrm{d}p} \\
&= \underbrace{\frac{\partial x_1^H}{\partial p} - \frac{\mathrm{d}x_1^S}{\mathrm{d}p}}_{<0} + \frac{\partial x_1^M}{\partial r}\underbrace{(x_1^S - x_1^M)}_{=-m_1}.
\end{aligned}$$

FYI: Slutzky Equation

Marshallian demand functions $x_j^M(p, y)$ solve the problem of utility maximization with a given budget y.

Hicksian demand functions $x_j^H(p, U)$ solve the problem of expenditure minimization with a given utility level U.

If both problems are based on the same utility function, we gain for an optimum

$$x_j^H(p, U) = x_j^M(p, e(p, U)) \quad \text{with} \quad y = e(p, U)$$

Partial differentiation with respect to p generates

$$\frac{\partial x_j^H}{\partial p} = \frac{\partial x_j^M}{\partial p} + \frac{\partial x_j^M}{\partial y}\frac{\partial e}{\partial p}.$$

Using Shephard's lemma (cf. p. 201) ($\partial e/\partial p = x_1^H = x_1^M$, where p is the relative price of the first good) we gain the Slutzky equation.

Oniki-Uzawa Offer Curves

Consequently

$$\frac{\mathrm{d}m_1(p)}{\mathrm{d}p} = \underbrace{\frac{\partial x_1^H}{\partial p} - \frac{\mathrm{d}x_1^S}{\mathrm{d}p}}_{<0} - \frac{\partial x_1^M}{\partial r} m_1 < 0 \quad \text{for} \quad m_1 = 0.$$

The Oniki-Uzawa offer curve has a negative slope at least for $m_1 = 0$.
Note for the autarky equilibrium $m_1(p^a) = 0$.
Moreover, the offer curve cannot cross the p-axis again; two zeros with a negative slope contradict a continuous curve.

By analogy, the foreign excess demand function $m_1^*(p)$ has the same properties. The above graph, however, uses a *mirrored ordinate*.

Oniki-Uzawa Offer Curves

The world excess demand

$$m_1^w(p) := m_1(p) + m_1^*(p)$$

decreases at p_e (with $m_1(p_e) = -m_1^*(p_e)$) if both countries have identical homothetic preferences (only one equilibrium possible):

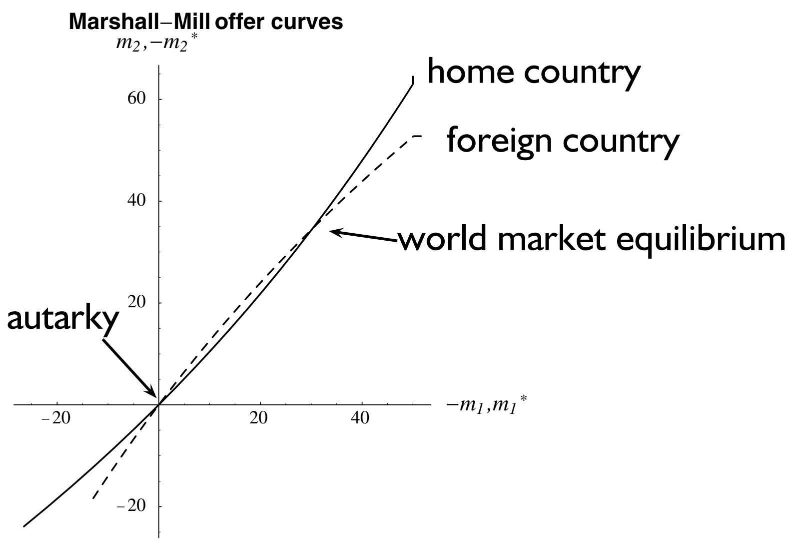
$$\frac{\mathrm{d}m_1^w(p_e)}{\mathrm{d}p} = \frac{\partial x_1^H}{\partial p} + \frac{\partial x_1^{H*}}{\partial p} - \frac{\mathrm{d}x_1^S}{\mathrm{d}p} - \frac{\mathrm{d}x_1^{S*}}{\mathrm{d}p} - \frac{\partial x_1^M}{\partial r} m_1 - \frac{\partial x_1^{M*}}{\partial r^*} m_1^*$$

$$\overset{m_1^*=-m_1}{=} \frac{\partial x_1^H}{\partial p} + \frac{\partial x_1^{H*}}{\partial p} - \frac{\mathrm{d}x_1^S}{\mathrm{d}p} - \frac{\mathrm{d}x_1^{S*}}{\mathrm{d}p} - \underbrace{\left(\frac{\partial x_1^M}{\partial r} - \frac{\partial x_1^{M*}}{\partial r^*}\right)}_{=0 \text{ for ident. hom. pref.}} m_1$$

$$= \frac{\partial x_1^H}{\partial p} + \frac{\partial x_1^{H*}}{\partial p} - \frac{\mathrm{d}x_1^S}{\mathrm{d}p} - \frac{\mathrm{d}x_1^{S*}}{\mathrm{d}p} < 0 \quad \text{for } m_1(p_e) + m_1^*(p_e) = 0.$$

Marshall-Mill Offer Curves

Alternative representation by Marshall-Mill offer curves. For any p the home country is willing to trade the excess supply $m_1(p) < 0$ against the excess demand $m_2(p) > 0$ at the world market (similarly for the foreign country). In autarky we have $m_1(p^a) = 0 = m_2(p^a)$.



International General Equilibrium (Free Trade)

Existence. For $m_1^w(p^a) > 0$ and $m_1^w(p^{a*}) < 0$ there must be some p_e with $p^a < p_e < p^{a*}$ such that $m_1^w(p_e) = 0$ (m_1 is continuous in p).

Stability. Suppose the following price formation process

$$m_1^w(p) \gtreqless 0 \implies \mathrm{d}p \gtreqless 0$$

Suppose also (has been derived before)

$$\frac{\mathrm{d}m_1^w(p_e)}{\mathrm{d}p} < 0 \quad \text{at an equilibrium}$$

We then find due to the continuity of $m_1^w(p)$:

$$\begin{array}{lccc} p = p^a \ (< p_e): & m_1^w > 0 & \rightarrow & \mathrm{d}p > 0 \\ p = p^{a*} \ (> p_e): & m_1^w < 0 & \rightarrow & \mathrm{d}p < 0 \end{array}$$

All price changes show a movement towards the equilibrium price p_e.

International General Equilibrium (Free Trade)

Uniqueness. If the world excess demand $m_1^w(p)$ decreases in every equilibrium and $m_1^w(p)$ is continuous in p, there can be only one equilibrium.

Summary. If both countries show identical homothetic preferences the equilibrium is unique and stable.

The equilibrium price ratio at the world market is located between the two autarky price ratios (except for extreme cases: strictly "between").

$$p^a < p_e < p^{a*}$$

Relative Price Advantages

Suppose autarky with two equilibria in the domestic and the foreign country. If both autarky price ratios coincide ($p^a = p^{a*}$) then

$$\underbrace{x_j^D = x_j^S}_{m_j=0} \quad \text{and} \quad \underbrace{x_j^{D*} = x_j^{S*}}_{m_j^*=0} \quad j = 1, 2.$$

Opening this situation for trade both countries stick to their former equilibrium at the same relative price; although trade is permitted both countries abstain from trade.

Condition for international trade. Foreign trade takes place only if there are different autarky price ratios in both countries ($p^a \neq p^{a*}$).

This argument loses significance in the case of increasing returns to scale.

Relative Price Advantages

Absolute vs. relative price advantages

Good 1 can be exported if autarky features

$$p_1^a < e p_1^{a*} \quad (e = \text{exchange rate } [€/\$])$$

so that the home country has an absolute price advantage.

The exchange rate is, therefore, one important determinant of absolute price advantages. A decline in the exchange rate can switch the absolute price advantage into a disadvantage.

(Regarding fixed exchange rate systems, devaluations with $e \uparrow$ are frequently discussed to support the domestic export industry.)

Relative Price Advantages

Suppose for a given exchange rate the home country would have also an absolute price advantage for good 2, it would try to export both commodities without imports. Trade in the sense of an exchange of goods would not happen and the balance of export and import value would be broken.

Solution. If the home country has an absolute price advantage for good 1, then the foreign country must have a corresponding advantage with regard to good 2. The home country imports good 2 if

$$p_2^a > e p_2^{a*}$$

Both conditions together can be reformulated as

$$\frac{p_1^a}{p_1^{a*}} < e < \frac{p_2^a}{p_2^{a*}}$$

(The direction of trade is reversed for opposite inequality signs.)

Relative Price Advantages

The exchange rate e adjusts between the above given autarky price ratios (disregarding other sources of trade in foreign exchange markets such as speculation). This statement is not too important regarding our real model without any exchange rate. The condition for trade including the assumed trade direction (export of good 1 and import of good 2) reduces, therefore, to

$$\frac{p_1^a}{p_1^{*a}} < \frac{p_2^a}{p_2^{*a}},$$

or

$$\frac{p_1^a}{p_2^a} < \frac{p_1^{*a}}{p_2^{*a}} \quad \Longleftrightarrow \quad p^a < p^{*a}$$

Foreign trade takes place if a country faces a relative or comparative price advantage in autarky.

Terms of Trade

Caveat. The dimension of a price ratio is

$$\frac{p_1}{p_2}\left[\frac{\frac{€}{\text{unit}_1}}{\frac{€}{\text{unit}_2}}\right] = \frac{p_1}{p_2}\left[\frac{\text{unit}_2}{\text{unit}_1}\right].$$

This trade relation is noted as terms of trade (*tot*). The *tot* in foreign trade (world market price ratio) denote how many units of the import good can be achieved by the home country for one unit of the export good. An increasing world market price ratio improves the real provision of the home country. Official statistics define however

$$tot := \frac{\text{price index of export goods}}{\text{price index of import goods}}.$$

(Hint: price indices are dimensionsless and so are the *tot*.)

Origins of Relative Price Advantages

Orthodox foreign trade theory assuming perfect competition is mostly dedicated to the following question: what are the origins of different autarky price ratios?
Regarding our theoretical body three causes are to be analyzed (cf. p. 26).

- different preferences (demand)
- different technologies (Ricardo model)
- different factor endowments (HOS model)

A simple explanation for different autarky price ratios and thus for foreign trade follows from different prefences.

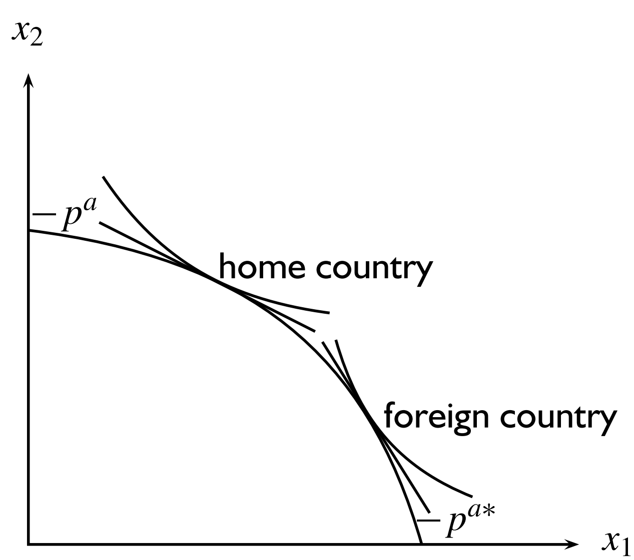
Assumption. Both countries are identical but for their preferences so that they have, in particular, identical transformation curves.

The following graph presents one transformation curve which is identical for both countries. The home country shows a relative stronger demand for good 2 compared to the foreign country.

The absolute slope of the price line is smaller at home.

$$p^a < p^{*a}$$

The home country exports, therefore, good 1 and imports good 2.



The following figure focuses just on the home country.

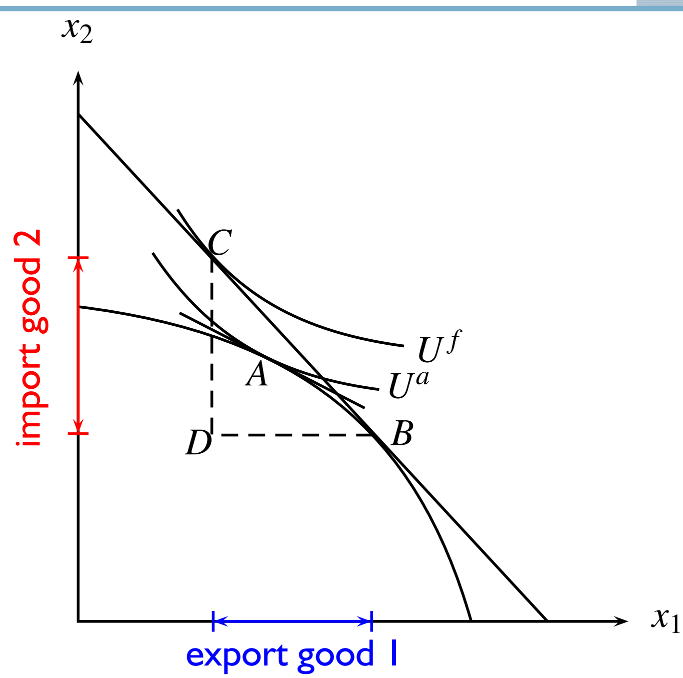
While production and consumption point coincide in autarky, foreign trade presents the option for differing points.

The free trade price ratio is located between the two autarky price ratios.

$$p^a < p < p^{*a}$$

This makes preferred consumption bundles feasible. The indifference curve in free trade exceeds the one in autarky ($\rightarrow$ gains from international trade).

A = production and consumption point in autarky
B = production point in free trade
C = consumption point in free trade
U^a = welfare in autarky
U^f = welfare in free trade
DC/DB = terms of trade



Assumptions and Key Result

Basic assumptions of the Ricardo model

- both countries differ in their production technologies
- you may assume identical preferences and factor endowments, but this is not necessary
- labor is the only factor of production
  (Ricardo: labor theory of value; today: simplification)
- constant labor coefficients

Key result. Any country exports the good where it has a comparative cost advantage. The other commodity is imported (theory of comparative cost advantages).

Domestic Production Sector (England)

The theory is presented by a numerical example (England and Portugal) and explained at the same time analytically.

England needs 10 units of labor to produce $1\,m^2$ of cloth (x_1) and 12 labor units to make $1\,l$ of wine (x_2). There are 220 labor units available (per unit of time).

Production functions

$$\text{cloth:} \quad x_1 = \frac{1}{a_1} v_1 = \frac{1}{10} v_1$$

$$\text{wine:} \quad x_2 = \frac{1}{a_2} v_2 = \frac{1}{12} v_2$$

where v_1 and v_2 denote the quantities of labor used in the production of cloth and wine, respectively.

Domestic Production Sector (England)

The fraction $x_1/v_1 = 1/10$ is called labor productivity and the inverse $v_1/x_1 = 10$ is referred to as labor coefficient in the production of cloth (similarly for the production of wine). More general, $1/a_1$ is the factor productivity and a_1 is the input coefficient of factor 1 in the production of good 1.

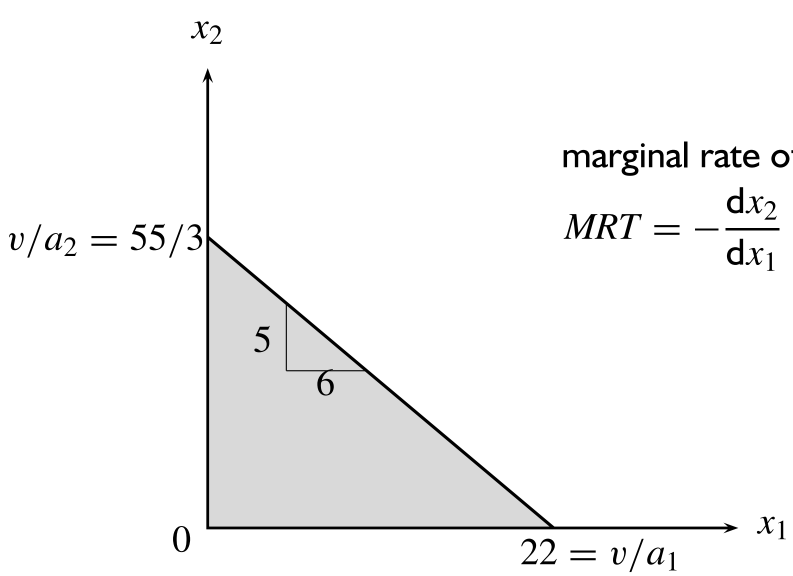
Available amount of labor

$$\begin{aligned} & v_1 + v_2 \leq v && \text{(factor consumption} \leq \text{endowment)} \\ \Longleftrightarrow\; & a_1 x_1 + a_2 x_2 \leq v && \text{(production functions)} \\ \Longleftrightarrow\; & 10x_1 + 12x_2 \leq 220 && \text{(numerical example)} \end{aligned}$$

For non-negative quantities of outputs $x_1 \geq 0$ and $x_2 \geq 0$ we gain England's production possibility set with the corresponding transformation curve.

$$x_2 = \frac{v}{a_2} - \frac{a_1}{a_2} x_1 = \frac{55}{3} - \frac{5}{6} x_1$$

Domestic Production Sector (England)



marginal rate of transformation:

$$MRT = -\frac{dx_2}{dx_1} = \frac{a_1}{a_2} = \frac{5}{6} = 0.8\bar{3}$$

Domestic Production Sector (England)

The MRT denotes how much wine England has to spare in order to produce one additional square meter of cloth or the cost of a square meter cloth in liters of wine. The opportunity cost of a square meter cloth gives $0.8\bar{3}\ l$ wine.

Definition

The opportunity cost of a good indicates the quantity of the other good that must be given up to achieve one additional unit of the good at hand.

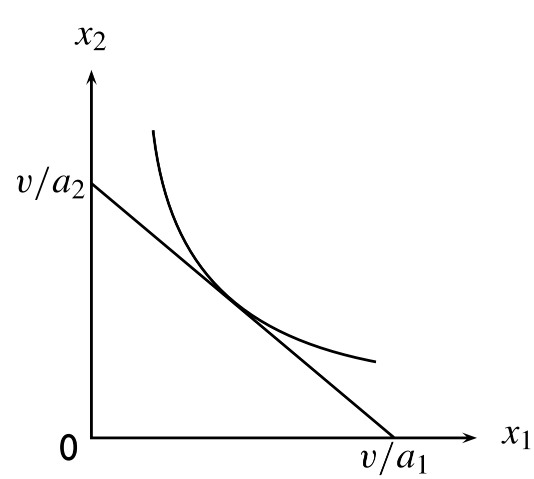
Except for corner solutions, the MRT corresponds to the relative price $p = p_1/p_2$. In our example the opportunity cost and the relative price p have the dimension l wine per m^2 cloth.

Autarky Equilibrium in England

As the MRT is constant in the Ricardo model, it suffices to know that both goods are consumed (no corner solution). The autarky price ratio is

$$p^a = \frac{a_1}{a_2} = \frac{5}{6} = 0.8\bar{3}$$

Caveat. Transformation curve and price line are identical, so you see only one line.



Foreign Production Sector (Portugal)

Portugal requires 9 labor units to produce $1\,m^2$ cloth and 6 labor units for $1\,l$ wine. 150 labor units are available in total.

In comparison to England, Portugal needs smaller quantities of labor for any unit of output. (Portugal shows smaller labor coefficients or higher labor productivities.) Thus, Portugal has absolute cost advantages in the production of both (!) commodities.

Question. Why should it be advantageous for Portugal to trade with England et vice versa?

Foreign Production Sector (Portugal)

Portugal's available amount of labor

$$a_1^* x_1^* + a_2^* x_2^* \le v^*, \qquad 9x_1^* + 6x_2^* \le 150$$

Portugal's transformation curve

$$x_2^* = \frac{v^*}{a_2^*} - \frac{a_1^*}{a_2^*} x_1^*, \qquad x_2^* = 25 - 1.5x_1^*$$

Portugal's opportunity cost for cloth

$$-\frac{\mathrm{d}x_2^*}{\mathrm{d}x_1^*} = \frac{a_1^*}{a_2^*} = 1.5$$

After all cloth x_1 is relative more costly in Portugal, although Portugal has an absolute advantage in the production of cloth.

$$p^{*a} = -\left.\frac{\mathrm{d}x_2^*}{\mathrm{d}x_1^*}\right|_{\text{Portugal}} = 1.5 > 0.8\bar{3} = -\left.\frac{\mathrm{d}x_2}{\mathrm{d}x_1}\right|_{\text{England}} = p^a$$

We say England has a comparative cost advantage in the production of cloth. As England's relative price of cloth in autarky is smaller than the corresponding price in Portugal ($p^a < p^{a*}$), we can state that England will export cloth (good 1) and import wine (good 2) after opening for trade.

More general (Ricardo theorem). Regarding international trade every country exports the good where it has a comparative advantage in production. The respective other good will be imported.

This principle of comparative advantages can be transferred to any other form of labor division.

England's comparative advantage in the production of cloth is reflected by Portugal's comparative cost advantage in the production of wine.

$$\frac{1}{p^{*a}} = -\frac{\mathrm{d}x_1^*}{\mathrm{d}x_2^*}\bigg|_{\text{Portugal}} = 0.\bar{6} < 1.2 = -\frac{\mathrm{d}x_1}{\mathrm{d}x_2}\bigg|_{\text{England}} = \frac{1}{p^a}$$

Portugal has to give up $0.\bar{6}\,m^2$ cloth to gain one additional liter of wine while England's waiver is $1.2\,m^2$ cloth.

Both countries thus can improve their welfare if they trade in accordance with the principle of comparative advantages (see also p. 101). Total production can be increased by specialization in those goods with comparative cost advantages (p. 99).

Comparative Advantages

Numerical example. Regarding autarky both countries can produce $10\,m^2$ cloth and $10\,l$ wine, respectively. Assume preferences such that these outputs are realised. In comparison to free trade total specialization would generate the following result.

	autarky		free trade	
	cloth	wine	cloth	wine
England	10	10	22	0
Portugal	10	10	0	25
world	20	20	22	25

Result. Both countries gain from international trade. Decisive factors are comparative rather than absolute advantages.

General Graphical Presentation

If both goods are consumed in both countries, the autarky price ratios are

$$\frac{a_1}{a_2} < \frac{a_1^*}{a_2^*} \iff p^a < p^{*a}.$$

Each country thus exports the commodity embodying a comparative advantage.

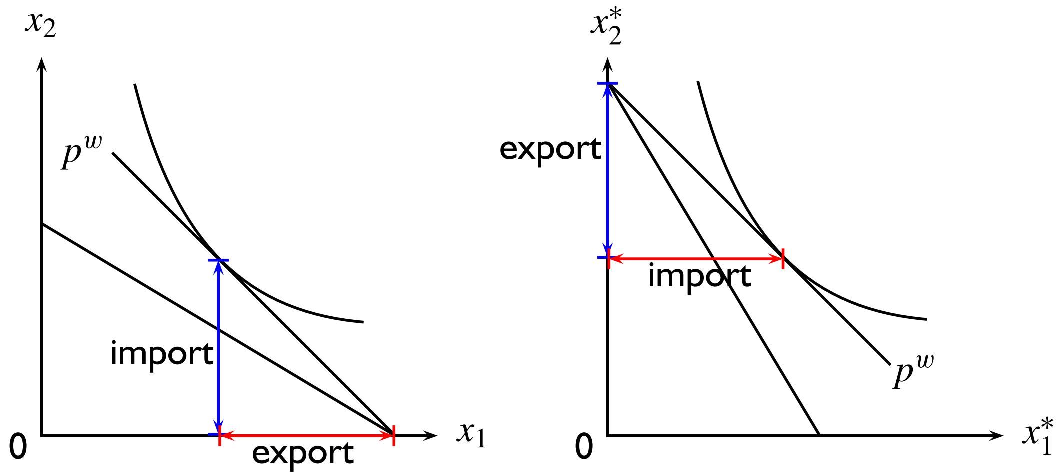
World market price ration p^w in free trade: $p^a \leq p^w \leq p^{*a}$. (This was shown with regard to the Oniki-Uzawa offer curves, which have to be adjusted slightly for the case of linear transformation curves.)

The following pages show the results after trade for both countries with regard to two cases.

$$\frac{a_1}{a_2} < p^w < \frac{a_1^*}{a_2^*} \quad \text{and} \quad \frac{a_1}{a_2} = p^w < \frac{a_1^*}{a_2^*}$$

General Graphical Presentation

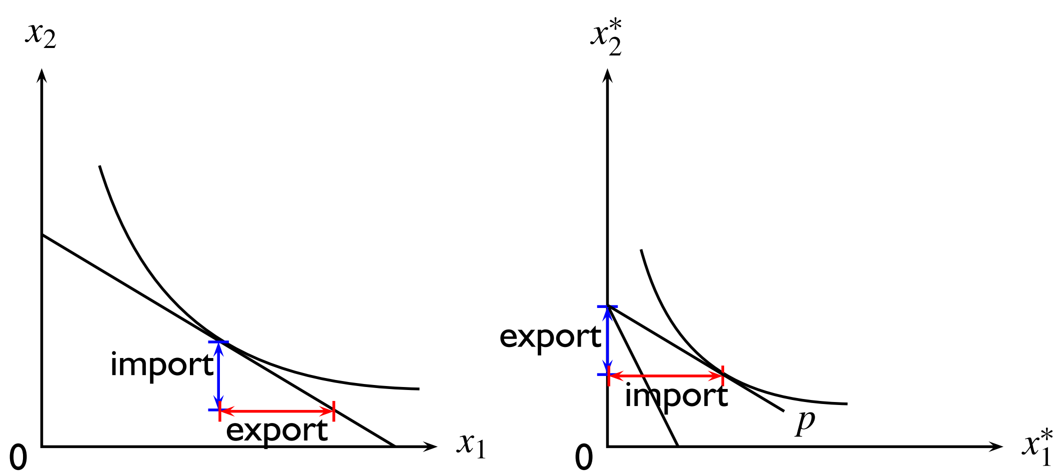
Total specialization of both countries



export good 1 = import good 1
import good 2 = export good 2

General Graphical Presentation

The case of a small country (without total specialization in the big country)



export good 1 = import good 1
import good 2 = export good 2

Big and Small Countries

The figures suggest that with an increasing difference in countries' sizes in particular smaller countries gain from trade.

This statement is only valid if big nations cannot exert market power via their trade policies.

In our setting, however, the small country has no effect on the world market price ratio, which corresponds to the autarky price of the other country (here $p^w = p^a$). Hence, only the small country can profit from price rearrangements.

This result corresponds to the statements at the beginning of the course. The relative importance of foreign trade increases the smaller the country is.

Total Specialization versus Diversification

The preceding Ricardo model suggests that at least one country tends to move towards total specialization of production. Open economies thus seem to produce only one commodity.

The reason lies in linear transformation functions. For strictly concave transformation functions, however, we observe usually only a partial specialization. Open economies produce more of the export good compared to autarky but they continue to produce both goods (diversification of production).

Final remark. Models with more than one factor of production und strictly concave transformation functions where comparative advantages are based on different total productivities may be seen as a generalization of the Ricardo model (cf. Markusen et al., Ch. 7).

Structure of the Model

The HOS model (Heckscher-Ohlin-Samuelson) traces differences in autarky price ratios (and thus foreign trade) back to differences in the relative factor endowments of the countries involved.

Basic assumptions. Both industries apply substitutional and linearly homogeneous production functions. The industrial specific functions do not have to be identical. We assume perfect competition and the factor endowments v_1 and v_2 are fixed. This setting holds true for both countries. $(f_1 \equiv f_1^*,\ f_2 \equiv f_2^*)$

In addition, both countries are characterized by identical homothetic preferences. Summing up, both countries differ merely by their relative abundance of factors of production ($\rightarrow$ factor-proportions model).

Result. The HOS theorem is stated on p. 127.

Unit Cost Functions at the Equilibrium

The analysis of the HOS model is greatly simplified by making use of unit cost functions. Profit maximizing firms have to minimize the respective unit cost.

Reminder. Cost functions on the basis of linearly homogeneous production function satisfy

$$c_j(q_1, q_2, x_j) = b_j(q_1, q_2)x_j.$$

Bear in mind that the unit cost functions

$$b_j(q_1, q_2) = \frac{c_j(q_1, q_2, x_j)}{x_j}$$

are independent of x_j.

Unit Cost Functions at the Equilibrium

The definition of unit cost functions yields also

$$b_j(q_1, q_2) = q_1 \frac{v_{1j}^D(q_1, q_2, x_j)}{x_j} + q_2 \frac{v_{2j}^D(q_1, q_2, x_j)}{x_j}$$
$$= q_1 a_{1j}(q_1, q_2) + q_2 a_{2j}(q_1, q_2)$$

where $a_{ij}(q_1, q_2)$ are the input coefficients (units of factor i per unit of good j). These coefficients are also independent of x_j for linearly homogeneous production functions. The optimal values of all input coefficients depend only on factor prices q_1 and q_2.

Shephard's lemma (p. 201) states

$$\frac{\partial c_j}{\partial q_i} = \frac{\partial b_j}{\partial q_i} x_j = v_{ij}^D \iff \frac{\partial b_j(q_1, q_2)}{\partial q_i} = \frac{v_{ij}^D}{x_j} = a_{ij}(q_1, q_2)$$

Unit Cost Functions at the Equilibrium

Note

$$b_j(q_1, q_2) > p_j : \quad \text{good } j \text{ not produced}$$
$$b_j(q_1, q_2) < p_j : \quad \text{positive profits, no equilibrium}$$

As a consequence any competitive equilibrium must hold

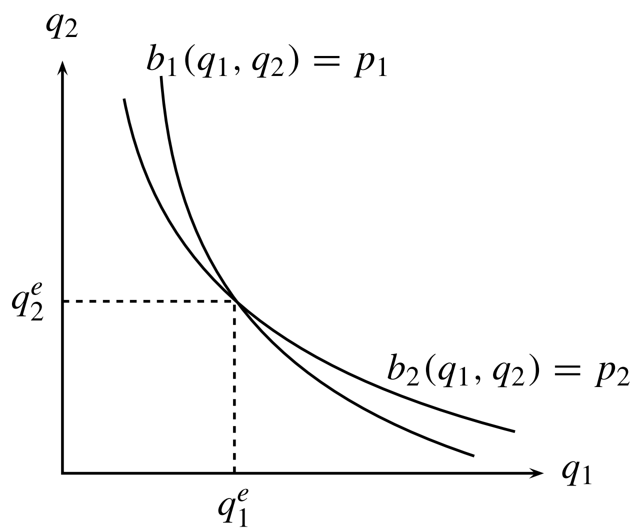
$$x_j > 0 \implies b_j(q_1, q_2) = p_j \quad \text{(zero profits).}$$

The subsequent figure depicts the two conditions

$$b_1(q_1, q_2) = p_1 \quad \text{and} \quad b_2(q_1, q_2) = p_2.$$

Unit Cost Functions at the Equilibrium

In an equilibrium good j is produced only if $b_j(q_1, q_2) = p_j$. Diversification with positive amount of both goods thus requires the factor prices (q_1^e, q_2^e). As long as both goods are produced the factor prices (q_1^e, q_2^e) are constant and hence the input cofficients $a_{ij}(q_1^e, q_2^e)$ must be constant.



3 Competitive General Equilibrium in Open Economies
3.5 The Heckscher-Ohlin-Samuelson Model

Unit Cost Functions at the Equilibrium

Diversification is assumed hereafter so that

$$x_1 > 0 \implies b_1(q_1^e, q_2^e) = p_1$$
$$x_2 > 0 \implies b_2(q_1^e, q_2^e) = p_2$$

Whether diversification prevails at given good prices depends in the end on the nation's factor endowments.

Remark. Given output prices (p_1, p_2) and assuming diversification before and after changes in the national factor endowments, the factor prices (q_1^e, q_2^e) and input coefficients remain $a_{ij}(q_1^e, q_2^e)$ constant. This statement becomes void if output prices are to be adjusted to the new situation.

Relative Factor Intensities

Definition

Good 1 uses factor 1 relatively intensively if

$$\frac{v_{11}}{v_{21}} = \frac{a_{11}(q_1, q_2)}{a_{21}(q_1, q_2)} > \frac{a_{12}(q_1, q_2)}{a_{22}(q_1, q_2)} = \frac{v_{12}}{v_{22}},$$

that is we use relatively more of input 1 in the production of one unit of good 1 than in the production of one unit of good 2.

In what follows we show that relative factor intensities can be determined by the shape of unit cost functions.

Relative Factor Intensities

Unit cost function of good j

$$b_j(q_1, q_2) - p_j = 0.$$

Using Shephard's lemma (p. 107)

$$\frac{\partial b_j(q_1, q_2)}{\partial q_i} = a_{ij}(q_1, q_2),$$

the slope of the unit cost functions follows from the implicit function theorem (p. 198):

$$-\left.\frac{\mathrm{d}q_2}{\mathrm{d}q_1}\right|_{\text{good j}} = \frac{\partial b_j(q_1, q_2)/\partial q_1}{\partial b_j(q_1, q_2)/\partial q_2} = \frac{a_{1j}(q_1, q_2)}{a_{2j}(q_1, q_2)}, \qquad j = 1, 2.$$

With reference to the preceding figure we depicted the curve for good 1 in the intersection point (q_1^e, q_2^e) steeper than the curve for good 2 so that

$$-\left.\frac{dq_2}{dq_1}\right|_{\text{good 1}} > -\left.\frac{dq_2}{dq_1}\right|_{\text{good 2}}$$

Applying the implicit function theorem this equivalent to

$$\frac{a_{11}(q_1^e, q_2^e)}{a_{21}(q_1^e, q_2^e)} = \frac{v_{11}}{v_{21}} > \frac{a_{12}(q_1^e, q_2^e)}{a_{22}(q_1^e, q_2^e)} = \frac{v_{12}}{v_{22}}.$$

By definition good 1 is relatively factor 1 intensive.

The inequality holds good for any factor allocation on the efficiency curve in the Edgeworth box on p. 35.

Theorem

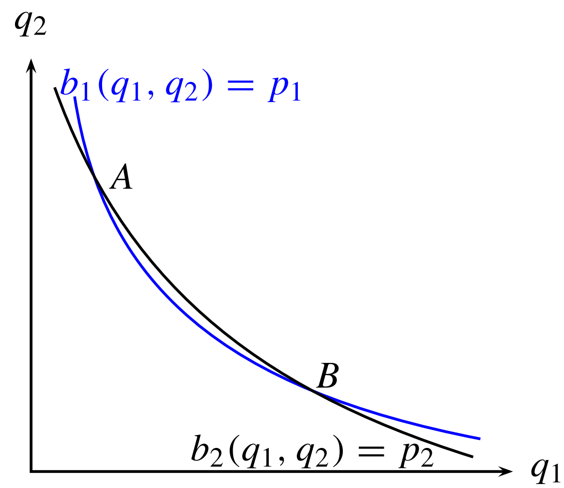
Consider the (q_1, q_2) diagram. If the unit cost function of good 1 runs steeper in the intersection point compared to the unit cost function of good 2 then good 1 uses factor 1 relatively intensively and good 2 is relatively factor 2 intensive.

From now on the conditions of this theorem will be assumed in general.

We exclude in particular the case of factor-intensity reversals, where more than one intersection point or factor price combination exists (any of these points would be compatible with diversification).

Factor-Intensity Reversals

A factor-intensity reversal implies at least two intersection points so that the factor price combinations (q_1, q_2) under diversification are not unique (either point A or B).
Diversification is compatible with two points where good 1 is relatively factor 1 intensive at A and relatively factor 2 intensive at B.



Comparative Statics at Diversification

What are the effects on the equilibrium in the production sector if we make changes in the (relative) factor endowments?

Full employment conditions

$$v_1 = v_{11} + v_{12} = a_{11}(q_1, q_2)\, x_1 + a_{12}(q_1, q_2)\, x_2$$
$$v_2 = v_{21} + v_{22} = a_{21}(q_1, q_2)\, x_1 + a_{22}(q_1, q_2)\, x_2$$

Assumption. A variation of v_1 and v_2 preserves the diversification of production. As long as the output prices do not respond, the factor prices (q_1^e, q_2^e) as well as the input coefficients $a_{ij}(q_1^e, q_2^e)$ remain constant.

Remark. At a later stage it is shown that the relative factor endowment is decisive for the choice of the production point. Starting at diversification small changes in the factor endowment usually do not offset diversification.

Comparative Statics at Diversification

For constant input coefficients ($a_{ij}(q_1, q_2) \rightarrow a_{ij}$) the total differential of the preceding system of equations yields

$$\mathrm{d}v_1 = a_{11}\,\mathrm{d}x_1 + a_{12}\,\mathrm{d}x_2$$
$$\mathrm{d}v_2 = a_{21}\,\mathrm{d}x_1 + a_{22}\,\mathrm{d}x_2,$$

Matrix form

$$\begin{pmatrix} \mathrm{d}v_1 \\ \mathrm{d}v_2 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} \mathrm{d}x_1 \\ \mathrm{d}x_2 \end{pmatrix} = \mathbf{A} \begin{pmatrix} \mathrm{d}x_1 \\ \mathrm{d}x_2 \end{pmatrix}$$

and if $\mathbf{A}$ is invertible (see below) then

$$\mathbf{A}^{-1} \begin{pmatrix} \mathrm{d}v_1 \\ \mathrm{d}v_2 \end{pmatrix} = \begin{pmatrix} \mathrm{d}x_1 \\ \mathrm{d}x_2 \end{pmatrix}$$

FYI: The Inverse of a Matrix

Given a 2×2 matrix $\mathbf{A}$, the inverse matrix $\mathbf{A}^{-1}$ is determined by

$$\mathbf{A}\,\mathbf{A}^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

The inverse $\mathbf{A}^{-1}$ exists if the determinant $|\mathbf{A}|$ is not zero. Then

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \qquad \mathbf{A}^{-1} = \frac{1}{|\mathbf{A}|} \begin{pmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{pmatrix}$$

and

$$|\mathbf{A}| = a_{11}a_{22} - a_{12}a_{21} \neq 0$$

On the invertibility of the matrix **A**

Assume good 1 to be relatively factor 1 intensive (cf. p. 111), that is

$$\frac{a_{11}}{a_{21}} > \frac{a_{12}}{a_{22}} \implies |\mathbf{A}| = a_{11}a_{22} - a_{12}a_{21} > 0.$$

With that the matrix **A** is invertible; the solution of the differentiated system of equations is

$$\begin{pmatrix} \mathrm{d}x_1 \\ \mathrm{d}x_2 \end{pmatrix} = \underbrace{\frac{1}{|\mathbf{A}|}\begin{pmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{pmatrix}}_{=:\mathbf{A}^{-1}} \begin{pmatrix} \mathrm{d}v_1 \\ \mathrm{d}v_2 \end{pmatrix}$$

3 Competitive General Equilibrium in Open Economies
3.5 The Heckscher-Ohlin-Samuelson Model
Comparative Statics at Diversification

Example. Increasing the available amount of factor 1, $\mathrm{d}v_1 > 0$, and holding factor 2 fixed, $\mathrm{d}v_2 = 0$, reveals

$$\frac{\mathrm{d}x_1}{\mathrm{d}v_1} = \frac{a_{22}}{|\mathbf{A}|} > 0, \quad \frac{\mathrm{d}x_2}{\mathrm{d}v_1} = \frac{-a_{21}}{|\mathbf{A}|} < 0.$$

This result is stated as

Theorem (Rybczinski theorem)

The production functions are linearly homogeneous and diversification prevails under the exclusion of factor-intensity reversals. For constant good prices and the increase of one factor quantity, the output of that good increases overproportionally *which uses the increasing factor relatively intensively. The output of the other good decreases.*

Remark: "overproportional"

For the sake of completeness it is to be shown that the increase of good 1 is overproportional. Note

$$v_1 = a_{11}x_1 + a_{12}x_2 > a_{11}x_1 \implies v_1/x_1 > a_{11}.$$

Multiplication by $\mathrm{d}x_1/\mathrm{d}v_1$ yields

$$\frac{\mathrm{d}x_1}{\mathrm{d}v_1}\frac{v_1}{x_1} > \frac{a_{22}}{|\mathbf{A}|}a_{11} = \frac{a_{11}a_{22}}{a_{11}a_{22} - a_{12}a_{21}} > 1 \quad \text{hence} \quad \frac{\mathrm{d}x_1}{x_1} > \frac{\mathrm{d}v_1}{v_1}.$$

Effects that follow from relative changes in factor endowments

Theorem

If good 1 is relatively factor 1-intensive then an increase of v_1/v_2 increases also the ratio x_1/x_2 (production point):

$$\frac{a_{11}}{a_{21}} > \frac{a_{12}}{a_{22}} \implies \frac{\mathrm{d}(x_1/x_2)}{\mathrm{d}(v_1/v_2)} > 0$$

Proof. Solving the conditions for full employment (without differentiation) for x_1 and x_2 we find

$$x_1 = \frac{1}{|\mathbf{A}|}(a_{22}v_1 - a_{12}v_2),$$
$$x_2 = \frac{1}{|\mathbf{A}|}(a_{11}v_2 - a_{21}v_1).$$

... to be continued

Comparative Statics at Diversification

Division of the two equations

$$\frac{x_1}{x_2} = \frac{a_{22}v_1 - a_{12}v_2}{a_{11}v_2 - a_{21}v_1} = \frac{a_{22}(v_1/v_2) - a_{12}}{a_{11} - a_{21}(v_1/v_2)}$$

Differentiation with respect to v_1/v_2 yields

$$\frac{\mathrm{d}(x_1/x_2)}{\mathrm{d}(v_1/v_2)} = \frac{|\mathbf{A}|}{\left[a_{11} - a_{21}(v_1/v_2)\right]^2}$$

This term is positive due to the assumption that good 1 is relatively factor 1-intensive (i.e. $|\mathbf{A}| > 0$). q.e.d.

Demand Side of the Model

For homothetic preferences the demand ratio is some function of p only (cf. p. 52)

$$x_1^D/x_2^D = g(p),$$

This ratio is in particular independent of the income (see again p. 52).

The general equilibrium before trade refers to a price ratio p^a such that $x_1^S = x_1^D$ and $x_2^S = x_2^D$. We therefore start with $x_1^S/x_2^S = x_1^D/x_2^D$.

An increase of v_1/v_2 in this equilibrium implies (cf. p. 77)

$$\frac{v_1}{v_2}\uparrow \quad \xrightarrow{p^a = \text{const.}} \quad \frac{x_1^S}{x_2^S}\uparrow \quad \longrightarrow \quad \frac{x_1^S}{x_2^S} > \frac{x_1^D}{x_2^D} \quad \longrightarrow \quad p = \frac{p_1}{p_2}\downarrow$$

An increase of v_1/v_2 thus lowers the domestic price ratio in autarky provided good 1 is relatively factor 1-intensive. (Note that the autarky equilibrium is stable.)

Two-Country Analysis

Assumptions for the factor-proportions theory

- identical homothetic preferences in both countries
- identical substitutional production functions for corresponding industries in both countries ($a_{ij} = a^*_{ij}$ for $i = 1, 2$ and $j = 1, 2$)
- different relative factor endowments

$$\frac{v_1}{v_2} > \frac{v_1^*}{v_2^*}$$

  (The home country is relatively v_1-abundant, the foreign country is relatively v_2-abundant),
- both countries show diversification of production in autarky
- good 1 is relatively factor 1-intensive, good 2 is relatively factor 2-intensive. Domestic and foreign country: $a_{11}/a_{21} > a_{12}/a_{22}$

Two-Country Analysis

For equal relative factor proportions the two countries would be identical and foreign trade would not take place.

$$\frac{v_1}{v_2} = \frac{v_1^*}{v_2^*} \implies p^a = p^{*a}$$

In the new setting we find

$$\frac{v_1}{v_2} \uparrow \implies \frac{v_1}{v_2} > \frac{v_1^*}{v_2^*} \quad \text{and} \quad p^a \downarrow \implies p^a < p^{*a}$$

The assumptions yield for the autarky price ratios with regard to the preceding results

$$p^a < p^{*a}.$$

The home country thus has a relative price advantage for good 1 and the foreign country for good 2, respectively. As a consequence regarding free trade the home country will export good 1 which uses the relatively abundant factor 1 more intensively. Good 2 will be imported. More generally we have:

Theorem (Heckscher-Ohlin theorem)

A country exports that good the production of which is relatively factor intensive with regard to the relatively abundant factor. The other good using relatively intensively the relatively scarce factor is imported.

Notes on the HOS model

- The HOS statement predicts that capital-abundant countries export capital-intensive goods and import labor-intensive goods (et vice versa). This hypothesis has been tested by Leontief for the presumed capital-abundant USA (1953).
  Empirical result. The USA export labor intensive goods and import capital intensive goods (Leontief paradoxon). Does this falsify the HOS theorem? No!
  Problems: missing data such as the factor intensities of export goods and, in particular, of import goods (one needs data of supplying countries) $\rightarrow$ auxiliary hypotheses (e.g., measurement of capital intensities of import goods via domestic substitutes etc.). Using improved auxiliary hypotheses and better data, more recent tests (since 1972) indicate results that support rather than falsify the HOS theorem.

Two-Country Analysis

- Extensions of the HOS model. Many factors of production, many goods (see Dixit/Norman), many countries, different types of goods (variants, qualities), different market structures etc. Nevertheless, the theoretical results tend to be reproducible.
- The HOS model is a static approach. Dynamic attempts might reveal additional aspects of international trade flows. Examples
  - dynamically increasing returns to scale (e.g., learning by doing); diminishing unit cost with increasing outputs over time $\rightarrow$ changing comparative advantages over time.
  - product cycles; establishment and spread of a new production processes (technical progress) or new products over time (innovation – export – imitation – import)

Two-Country Analysis

Remark. The Rybczinski theorem and the Heckscher-Ohlin theorem as well as the subsequent theorem on the equalization of factor prices and the Stolper-Samuelson theorem form the so-called four core theorems of the HOS model.

Effects on Factor Prices

Effects on factor prices. (1) What happens to factor prices if output prices are changed by foreign trade (Stolper-Samuelson)? (2) Assuming internationally immobile factors of production, how can factor prices be equalized simply by trade in goods?

Ad (1) Determination of Stolper-Samuelson derivatives $\mathrm{d}q_i/\mathrm{d}p_j$

Unit cost-price-restrictions for diversification

$$p_j = b_j(q_1^e, q_2^e), \qquad j = 1, 2.$$

Their total differentials

$$\begin{aligned}\mathrm{d}p_1 &= \frac{\partial b_1(q_1^e, q_2^e)}{\partial q_1}\mathrm{d}q_1 + \frac{\partial b_1(q_1^e, q_2^e)}{\partial q_2}\mathrm{d}q_2 \\ \mathrm{d}p_2 &= \frac{\partial b_2(q_1^e, q_2^e)}{\partial q_1}\mathrm{d}q_1 + \frac{\partial b_2(q_1^e, q_2^e)}{\partial q_2}\mathrm{d}q_2\end{aligned}$$

Effects on Factor Prices

Using Shephard's lemma (p. 107) yields

$$\begin{aligned}\mathrm{d}p_1 &= a_{11}(q_1^e, q_2^e)\mathrm{d}q_1 + a_{21}(q_1^e, q_2^e)\mathrm{d}q_2 \\ \mathrm{d}p_2 &= a_{12}(q_1^e, q_2^e)\mathrm{d}q_1 + a_{22}(q_1^e, q_2^e)\mathrm{d}q_2,\end{aligned}$$

or in matrix notation

$$\begin{pmatrix}\mathrm{d}p_1 \\ \mathrm{d}p_2\end{pmatrix} = \begin{pmatrix}a_{11} & a_{21} \\ a_{12} & a_{22}\end{pmatrix}\begin{pmatrix}\mathrm{d}q_1 \\ \mathrm{d}q_2\end{pmatrix} = \mathbf{B}\begin{pmatrix}\mathrm{d}q_1 \\ \mathrm{d}q_2\end{pmatrix}$$

Regarding the former $\mathbf{A}$ we have now $\mathbf{B} = \mathbf{A}^\mathsf{T}$

Effects of changes in output prices on factor prices (if $\mathbf{B}$ is invertible)

$$\mathbf{B}^{-1}\begin{pmatrix}\mathrm{d}p_1 \\ \mathrm{d}p_2\end{pmatrix} = \begin{pmatrix}\mathrm{d}q_1 \\ \mathrm{d}q_2\end{pmatrix}$$

Effects on Factor Prices

Assume again that good 1 is relatively factor 1-intensive then

$$\frac{a_{11}}{a_{21}} > \frac{a_{12}}{a_{22}} \implies |\mathbf{B}| = a_{11}a_{22} - a_{12}a_{21} > 0.$$

The matrix **B** is thus invertible. The solution of the differentiated system of equations is

$$\begin{pmatrix} \mathrm{d}q_1 \\ \mathrm{d}q_2 \end{pmatrix} = \frac{1}{|\mathbf{B}|} \begin{pmatrix} a_{22} & -a_{21} \\ -a_{12} & a_{11} \end{pmatrix} \begin{pmatrix} \mathrm{d}p_1 \\ \mathrm{d}p_2 \end{pmatrix}$$

Example. An increase of the price of the first good, $\mathrm{d}p_1 > 0$, with a constant price of the other good, $\mathrm{d}p_2 = 0$, yields

$$\frac{\mathrm{d}q_1}{\mathrm{d}p_1} = \frac{a_{22}}{|\mathbf{B}|} > 0, \quad \frac{\mathrm{d}q_2}{\mathrm{d}p_1} = \frac{-a_{12}}{|\mathbf{B}|} < 0.$$

Effects on Factor Prices

A comparison with the results computed for the Rybczinski derivatives shows that the increase of p_1 is overproportional in relation to the decrease of p_2.

Theorem (Stolper-Samuelson Theorem)

The production functions are linearly homogeneous and diversification of production without factor-intensity reversals prevails. An increase of the price of a good increases overproportionally the price of the production factor which is used relatively intensively in that industry. The price of the other factor is reduced.

Effects on Factor Prices

- The Stolper-Samuelson theorem explains how factor prices vary in the sequence of commodity price changes.
- These shifts in the transition to free trade can also be caused by other reasons such as tariffs or taxes.
- Moreover, factor price changes influence the income distribution.

Example. The transition to free trade increases in a labor-abundant country the relative price of the labor-intensive good 1 (export good). As a consequence of the Stolper-Samuelson theorem the wage rate q_1 increases while the rental rate for capital q_2 is reduced. For fixed factor endowments the labor income $q_1 v_1$ rises and the capital income $q_2 v_2$ declines. The functional income distribution is shifted in favor of the workers.

Effects on Factor Prices

Ad (2) Factor price equalization

Assumptions

- Both countries have identical linearly homogeneous production functions per industry.
- The countries differ only by their factor endowments (and possibly by their preferences).
- The free trade equilibrium shows a unique price ratio $p = p_1/p_2$ for both countries.
- Diversification of production prevails in both countries.

Then both countries show the same unit cost constraints expressed in units of good 2 (real factor prices: $\tilde{q}_i = q_i/p_2,\ i = 1, 2$).

(3)
$$\begin{aligned} b_1(\tilde{q}_1, \tilde{q}_2) &= p \\ b_2(\tilde{q}_1, \tilde{q}_2) &= 1 \end{aligned}$$

Note that differing factor endowments do not affect the unit cost-price relations.

The problem of factor price equalization is therefore reduced to the question if the system (3) can uniquely be solved for $(\tilde{q}_1, \tilde{q}_2)$.

Jacobi matrix with respect to $(\tilde{q}_1, \tilde{q}_2)$ (note again Shephard's lemma):

$$\mathbf{B} = \begin{pmatrix} a_{11}(\tilde{q}_1, \tilde{q}_2) & a_{21}(\tilde{q}_1, \tilde{q}_2) \\ a_{12}(\tilde{q}_1, \tilde{q}_2) & a_{22}(\tilde{q}_1, \tilde{q}_2) \end{pmatrix}$$

The implicit function theorem states that system (3) has a locally unique solution $(\tilde{q}_1, \tilde{q}_2)$ if $|\mathbf{B}| \neq 0$. Samuelson and Gale/Nikaido have proved the system has a globally unique solution if $|\mathbf{B}|$ does not change its sign for all non-negative values of $\tilde{q}_1, \tilde{q}_2$ (and if $a_{11}a_{22} \neq 0$ for all non-negative $\tilde{q}_1, \tilde{q}_2$).

This is the case if factor-intensity reversals do not occur. Regarding a relative factor 1-intensive good 1 the following inequality holds true for all $(\tilde{q}_1, \tilde{q}_2) \geqq (0, 0)$

$$\frac{a_{11}(\tilde{q}_1, \tilde{q}_2)}{a_{21}(\tilde{q}_1, \tilde{q}_2)} > \frac{a_{12}(\tilde{q}_1, \tilde{q}_2)}{a_{22}(\tilde{q}_1, \tilde{q}_2)} \quad \Longleftrightarrow$$
$$|\mathbf{B}| = a_{11}(\tilde{q}_1, \tilde{q}_2)a_{22}(\tilde{q}_1, \tilde{q}_2) - a_{21}(\tilde{q}_1, \tilde{q}_2)a_{12}(\tilde{q}_1, \tilde{q}_2) > 0$$

If these conditions are satisfied, the real factor prices will be equalized in both countries.

Theorem (Factor price equalization theorem)

If factor-intensities do not reverse, the assumptions made suffice for an equalization of factor prices.

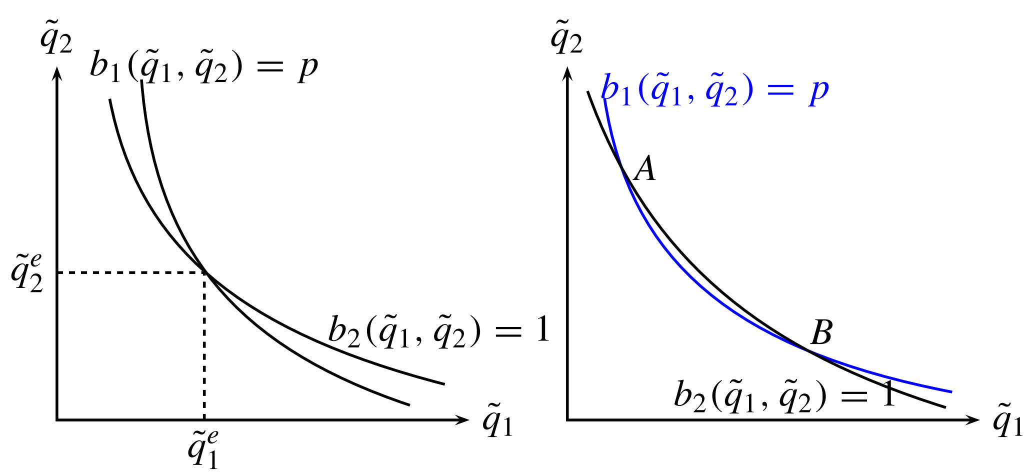
Effects on Factor Prices

Allowing for factor-intensity reversals different factor price relation can prevail even if diversification holds true in both countries.

The following page shows two well known figures now with real factor prices $\tilde{q}_1$ and $\tilde{q}_2$.

- The left figure excludes factor-intensity reversals so that factor price equalization (i.e. $\tilde{q}_1^e$ and $\tilde{q}_2^e$) takes place for diversification.
- The right figure shows diversification for the two points A and B The system (3) has no unique solution. Factor price equalization may, but does not have to occur.

3 Competitive General Equilibrium in Open Economies
3.5 The Heckscher-Ohlin-Samuelson Model
Effects on Factor Prices



Gains from Trade and Specialization

The addressed gains from foreign trade will now be presented systematically.

In principle there are two sources of profits.

- Gains from trade. The consumer is now able to realize the autarky utility level at foreign trade prices more efficiently. The additional trade possibilities permit a higher utility level.
- Gains from specialization. The producers realize a higher production value at foreign trade prices compared to autarky outputs.

Caveat. We discuss here the case of one representative consumer. Generalizations are possible, but for several persons we have to pay attention to losers beside winners (compensation problem).
Example. Stolper-Samuelson theorem and income (re-)distribution.

Gains from Trade

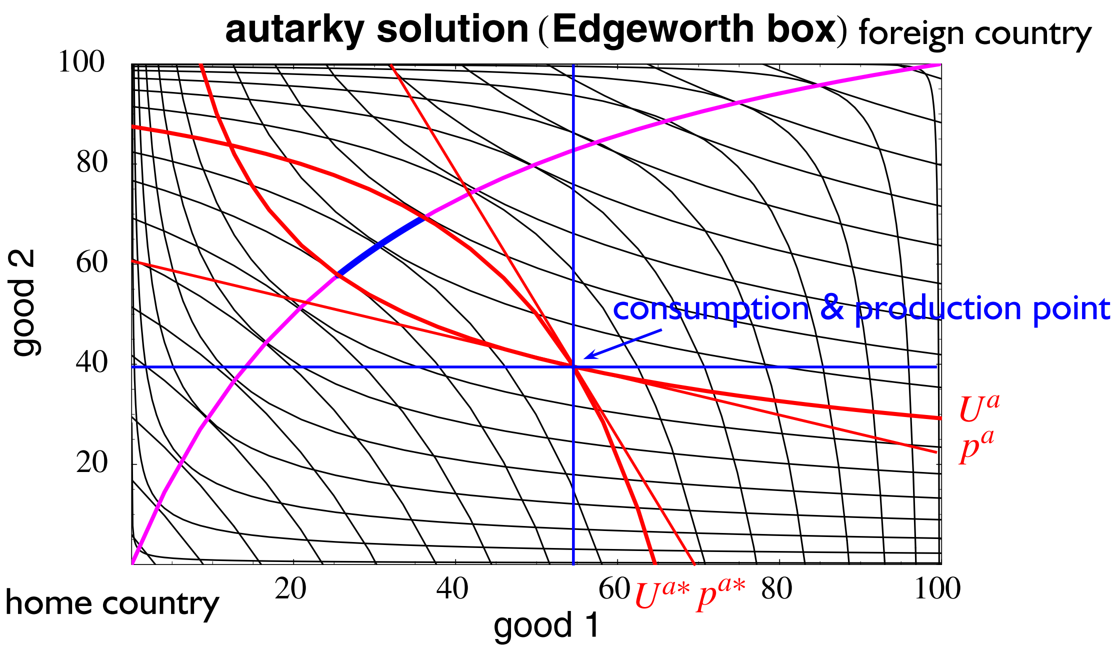
Gains from trade result merely from international trade acts.
In order to exclude gains from specialization let us fix the autarky factor allocation (completely immobile production factors on a national level). This fixes the production point (given x_1 and x_2) and the (blue) transformation curves are reduced to rectangles.

Production and consumption points coincide in both countries (no trade in autarky of course).

Global production of both goods corresponds to the width and height of the following Edgeworth box.

Gains from Trade

national general equilibria in closed economies ($p = p_1/p_2$)



Gains from Trade

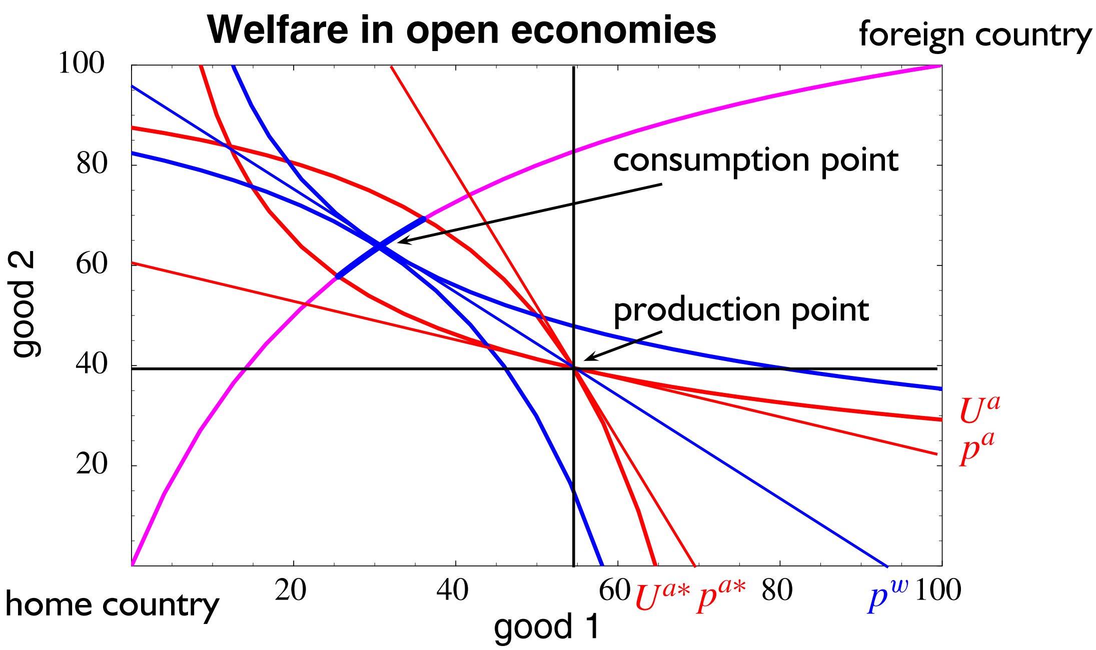
The national welfare in autarky is depicted by (red) indifference curves U^a and U^{a*}. The corresponding price relations are p^a and p^{a*}. Note for rectangular transformation curves (supply side) that the price relations are solely determined by the slope of the corresponding indifference curves (demand side).

(magenta) contract curve geometric location of all tangent points between domestic and foreign community indifference curves ($\rightarrow$ Pareto optima).

(blue) core: the part of the contract curve where both countries attain at least their autarky utility level.

The world market price ratio p^w lies between p^a and p^{a*}. The consumption point has to be in the core as the world market price ratio is valid for both countries (tangent point).

Gains from Trade



Gains from Trade

Welfare in open economies. Foreign trade generates for at least one country advantages.

Both countries profit if p^w lies between p^a and p^{a*}. The world market price ratio p^w determines the new consumption point (tangent of (blue) indifference curves with higher levels than in autarky for both countries). Hence, *both countries gain from trade*.

Summary. The production points remain fixed for both countries by assumption, but new consumption points for both countries on the contract curve.

home country	export good 1 = import good 1		foreign country
	import good 2 = export good 2		

Gains from Specialization

Gains from specialization result from a reorganization of national factor assignments to production industries (inter-industrial reallocation of factors).

Theorem

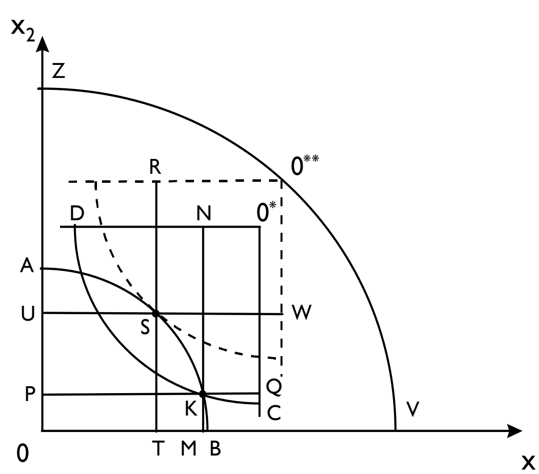
Assuming diversification and strictly concave transformation curves, the optimum allocation of production factors is achieved if the marginal rates of transformation are equalized between both countries:

$$\left|\frac{\mathrm{d}x_2}{\mathrm{d}x_1}\right| = \left|\frac{\mathrm{d}x_2^*}{\mathrm{d}x_1^*}\right| = p^w$$

We explain this theorem now graphically.

Gains from Specialization

Regarding autarky the domestic production block is $0AB$ and the foreign block is 0^*CD.
If K is the common production point for closed economies then total production of good 1:
$PK + KQ = PQ$
total production of good 2:
$KM + KN = MN$



Gains from Specialization

Koopmans efficiency. Given factor endowments and given production technologies in both countries, a production point is said to be *Koopmans efficient* if there is no way to increase one global output without decreasing the other one.

Point K thus cannot be Koopmans efficient. A shift of the foreign production block to north-east ($O^* \rightarrow O^{**}$) until we find the tangent point S increases both outputs:
total production of good 1: $UW > PQ$
total production of good 2: $TR > MN$

3 Competitive General Equilibrium in Open Economies
3.6 Gains from International Trade

Gains from Specialization

Point S is Koopmans efficient as the global output of one good can only be increased by a reduction of the other output. In accordance with the theorem above, notice that the national marginal rates of transformation are equalized in S.

Alternative Koopmans efficient points follow from moving the foreign production block tangentially along the domestic production block. These points including O^{**} describe the world transformation curve ZV.

Gains from specialization. Free trade generates a common world market price ratio. All industries adjust their marginal rate of transformation to the (inverse) world market price ratio so that the rates of transformation are equalized. The new solution must, therefore, be Koopmans efficient.

Gains from Specialization

- Koopmans efficiency means that the global output of one good is maximized holding the other output fixed. Note that all factors of production are only mobile on a national level. If a factor price equalization takes place even an international reallocation of ressources cannot increase global outputs anymore.
- Identifying point S with the production point in the figure before we can depict combined gains from trade and specialization in just one figure (see below).
- Note finally that we analyze here just total gains from trade but not their sharing to countries.

Total Gains of a Country

The total gains of a country *in monetary terms* can be depicted in a commodity space.

Using hypothetical budget constraint it is possible to distinguish between gains from trade and gains from specialization.

Start with general equilibrium of a closed country and the welfare level U^a (the autarky price ratio is omitted).

Switching to free trade changes the autarky price ratio which results here in a steeper revenue (or budget) line.

Total Gains of a Country

Assume some world market price ratio p^w consistent with a general equilibrium for an open economy.

$y^1 =$ minimum income to realize U^a at p^w

$y^2 =$ income at p^w without specialization

gains from trade $\hat{U} - U^a$

$y^3 = r(p^w)$ revenue and income at p^w after specialization

gains from specialization $U^f - \hat{U}$

total gains $U^f - U^a$

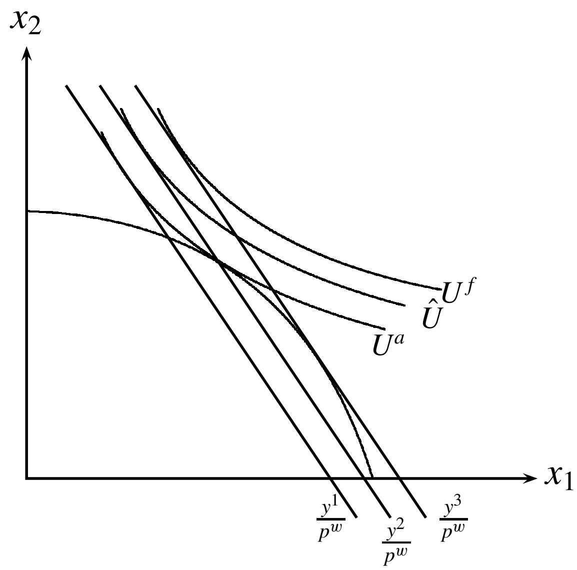


Table of Contents

Market Structures and Competition

Structural characteristics of a perfectly competitive market

- numerous buyers and sellers (all acting as price takers)
- homogeneous products (not differentiated or heterogeneous)
- free and open markets (no barriers to entry or exit, no transport cost)
- rational actors (which do not discriminate unfairly)
- perfect information
- non-increasing returns to scale, no externalities

Regarding perfectly competitive markets we find the law of undifferentiated prices (or unique world market prices).

Perfect competition is used as reference model for market economies. All agents act as price takers without market power (no abuse of market power possible) and they do not discriminate neither at home nor the rest of the world.

Market Structures and Competition

Imperfect competition exists whenever at least one condition for perfect competition fails to hold good.
Problem. There are numerous possible constellations of imperfect competition.

Research in foreign trade theory is, therefore, focused on homogeneous and heterogeneous oligopolies as well as monopolistic competition (competition with heterogeneous goods).
Moreover, regional economics is particularly interested in markets which are not simply point-shaped ($\rightarrow$ transport cost).

Trade Policy Regarding Perfect Competition

Free trade theorem. With regard to perfect competition free trade is always favorable against autarky due to gains from trade (at least if all losers are compensated).

Is it possible though to explain existing trade restrictions such as import tariffs within this model of perfect competition?

Frequently discussed arguments

- If losers (such as competitors in the import market) are not compensated after the transition to free trade we can expect that interest groups will be formed in order to take influence on political decision makers.
  Problem. Pursuing particular interests damages usually general public.

Trade Policy Regarding Perfect Competition

- Big countries have – at least up to a certain degree – market power on the world markets. These countries are able to influence world market prices in their own favor by charging certain tariffs (terms of trade argument, theory of optimal tariffs). Such tariffs increase the national welfare compared to autarky as well as free trade.
  Problem. Foreign retaliation measures; trade wars will damage all participating countries.
- Infant industry argument. It can be useful in the absence of perfect competition to protect young industries such that they reach international competitiveness, e.g., by learning effects.
  Problem. What industries are really worthy of protection? Protection should be a temporary measure! Here, subsidies are superior to tariffs.

Trade Policy Regarding Perfect Competition

Besides the above points further arguments against free trade follow purely protective directions such as security of supply of certain goods (foods, energy) or security of domestic jobs.

In view of the traditional trade theory the infant industry argument turns out to be the only solid point against free trade and for temporary subsidies of young industries.

Regarding more recent trade theories supposing *imperfect competition* the statement in favor of free trade is weaker. This may be seen as a starting point for a strategic trade policy.

Welfare Effects of a Tariff

Example. Protection by a tariff for an import market (or assistance by a subsidy on an export market $\rightarrow$ dumping, p. 177)

- firms profit (higher prices and profits per unit at home)
  indirect effects on prices of intermediate goods are ignored
- households lose (cheaper imports no longer available)
- governments tend to play that game due to customs revenues

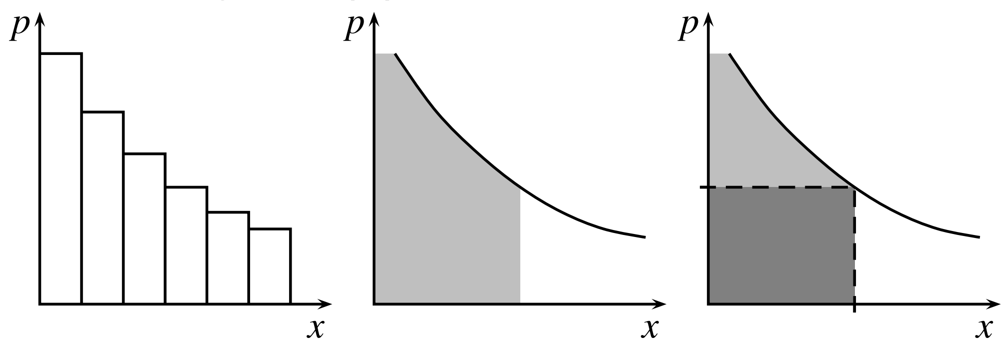
Caveat. The strategy is usually performed under the pretence to protect jobs, but the economy as a whole suffers a net loss in welfare.

Here partial equilibrium model. The analysis of counter-measures requires a general equilibrium model including effects in the export market.

FYI: Consumer Surplus

The consumer surplus denotes the difference between the **consumers willingness to pay** and the actual **consumption expenditure**.

The willingness to pay is computed on the basis of the demand curve (marginal willingness to pay). We assume that each unit (1st, 2nd, 3rd ...) of the commodity can be sold at the highest possible price the consumers are willing to pay (perfect price differentiation). As a consequence the area (integral) below the demand curve represents the total willingness to pay of the consumers.

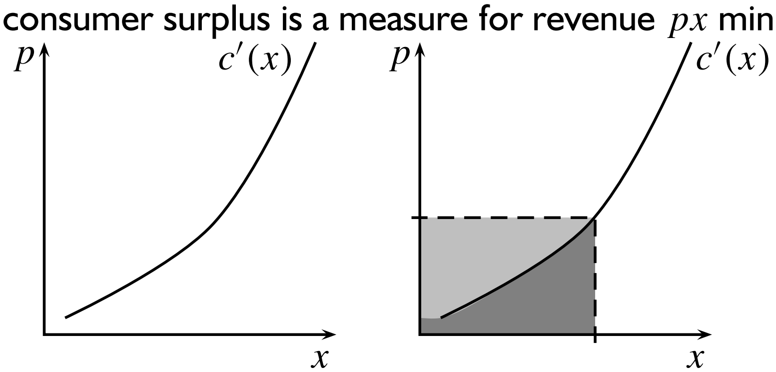


FYI: Producer Surplus

The producer surplus refers to marginal cost pricing. The supply function is the reverse of $p = c'(x)$. The area below the supply curve indicates therefore variable cost.

$$\int_0^x c'(x)\mathrm{d}x = c(x) - c(0)$$

Given x, all consumers pay the same price p. Consequently, the consumer surplus is a measure for revenue px minus (variable) cost.



Welfare Effects of a Tariff

The case of a *small* country

price effect:

$p^w \rightarrow p^c = p^w(1+t)$

consump. effect: $x^D \rightarrow x^{D\prime}$

protective effect: $x^S \rightarrow x^{S\prime}$

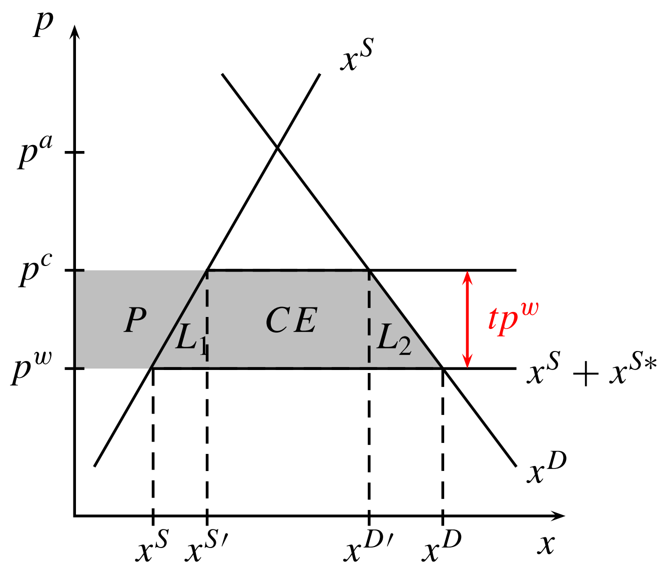
trade effect:

$(x^D - x^S) \rightarrow (x^{D\prime} - x^{S\prime})$

customs effect: CE

redistribution:

- producer surplus P
- consumer surplus $P + L_1 + CE + L_2$
- deadweight loss $L_1 + L_2$



Economies of Scale

One of the most important reasons for imperfect competition are increasing returns to scale and corresponding decreasing cost per unit.

A production function $x = f(v_1, v_2)$ exhibits globally increasing returns to scale if the scale elasticity $(\mathrm{d}x/x)/(\mathrm{d}\lambda/\lambda)$ exceeds always one. (A proportional multiplication of inputs by some factor λ implies an overproportional increase of the output (factor $\mu > \lambda$)).

The unit cost thus decreases with the output

$$\frac{c(x)}{x} = \frac{q_1\hat{v}_1 + q_2\hat{v}_2}{x} \longrightarrow \frac{q_1\lambda\hat{v}_1 + q_2\lambda\hat{v}_2}{\mu x} < \frac{c(x)}{x}$$

One can show that increasing returns to scale prevail (at an efficient use of inputs) if and only if unit cost c/x exceeds marginal costs c'.

Perfect competition with marginal cost pricing ($c'(x) = p$) at a profit maximum is, therefore, incompatible with increasing returns to scale: $c(x)/x > c'(x) = p$ indicating a loss.
Moreover, each firm would persue an increasing market share to exploit decreasing unit cost.

Example. The cost function $c(x) = a + bx$ with $a, b > 0$ shows decreasing unit cost. The inequality

$$\frac{c(x)}{x} = \frac{a}{x} + b > b = c'(x)$$

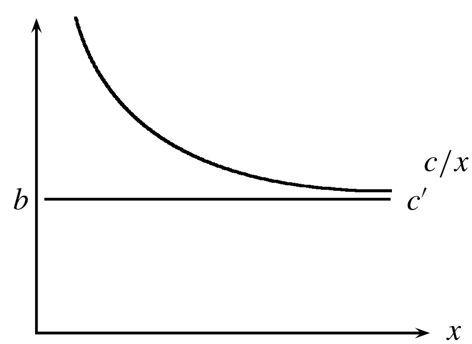
tells us that the corresponding production function exhibits increasing returns to scale.

Fixed cost a are typical for network industries with big capital stocks or can be caused, e.g., by R&D activities.

Example for decreasing unit cost:

$$c(x) = a + bx, \; c'(x) = b,$$
$$\frac{c(x)}{x} = \frac{a}{x} + b$$

Thus $c/x > c'$ for all x.



Summary. If two closed countries show a monopoly then – regarding a transition to free trade – the bigger monopolist can operate at lower unit cost and consequently he will eliminate his rival.
Advantage. Lower consumer prices in the absence of market power.
Disadvantage. The smaller monopolist has to leave the market (and the survivor realizes market power).

Basic Arguments of Strategic Trade Policy

An argument of strategic trade policy presented by Brander and Spencer refers to the assumption that two big firms residing in different countries compete with each other ($\rightarrow$ duopoly).

Making use of subsidies to promote the domestic duopolist might lead to an equilibrium preferable for the home country.

The reasoning can be demonstrated by a simple, static game theoretical model. This requires some basics in game theory.

Game Theoretic Basics

The strategic form of two-person games appears as a payoff matrix. Each player has a set of strategies and the payoff matrix shows for every combination of strategies the payoffs for both players.

In our case player A has the strategies *top* and *bottom*, while player B can chose *left* or *right*. Every field in the payoff matrix (next page) shows a pair of payoffs; the left value refers to player A and the right value to player B.

Due to its original interpretetation the example given below is called the prisoners' dilemma. The non-cooperative (or competitive) solution is relatively simple as both players have dominant strategies.

Game Theoretic Basics

Independent of the decision of player B, it is always advantageous for player A to chose the strategy *bottom* (dominant strategy). Similarly, *right* is the dominant strategy for player B.

		player B	
		left	right
player A	top	(3,3)	(0,4)
	bottom	(4,0)	(1,1)

In this game the strategy combination (*bottom*,*right*) is a non-cooperative equilibrium. Be aware that the total payoff $(1 + 1 = 2)$ is smaller than for (*top*,*left*) with $3 + 3 = 6$ which would be feasible if both players cooperate ($\rightarrow$ prisoners' dilemma).

4 International Trade under Imperfect Competition
4.2 Trade Policy under Perfect and Imperfect Competition

Game Theoretic Basics

The solution of the previous game is called a Nash equilibrium which can also exist if there are no dominant strategies.

Definition

A Nash equilibrium is a situation where no single player can obtain a higher payoff by changing his own strategy, if the others stick to their strategies. (No player can profit from leaving an equilibrium if the others do not move.)

The following example includes no dominant strategy, but it has two Nash equilibria (mutually best answers to given strategies of the others).

Game Theoretic Basics

If player A supposes that B plays *left* then A choses *top*. If player B expects A to chose *top*, he answers *left*. If one player leaves (*top*,*left*) his payoffs would be reduced. We have found a Nash equilibrium.
Similarly, the pair (*bottom*,*right*) determines a second Nash equilibrium.

		player B	
		left	right
player A	top	(2,1)	(0,0)
	bottom	(0,0)	(1,2)

Substituting the payoffs for (*top*,*left*) in this game by $(-1, 1)$, we find a game with a unique Nash equilibrium and no dominant strategy for player B.

Game Theoretic Basics

The problem of ambiguous or non-existing Nash equilibria in pure strategies requires more knowledge about game theory (mixed strategies, refinements of Nash equilibria, dynamic games, etc.) which goes beyond the scope of this course.

The presented concepts suffice, however, for our purpose to explain basic arguments of strategic trade policy.

Strategic Trade Policy

Suppose two companies originated in different countries plan to produce a commodity which induces substantial R&D cost.
Example. Boeing versus Airbus (747x and A 380). World market 1997: Airbus 1650 planes, Boeing 7000 planes, and others 600 planes).

If both firm provide the good both operate at a loss (price of sale and number of units are too small). If only one of them produces the good decreasing unit cost yields a considerable profit.

This situation can be described by the following payoff matrix.

Strategic Trade Policy

The payoff matrix shows two Nash equilibria (*P*,*NP*) and (*NP*,*P*). Which equilibrium will be attained depends on the specific situation. For instance, which company is more credible to start the production effectively (bigger, older, reputation).

		Boeing: P	Boeing: NP
Airbus	P	(−10,−10)	(100,0)
	NP	(0,100)	(0,0)

As the expected profit is remarkable both economies are strongly interested in holding a global player (→ jobs, export, market power). With regard to strategic trade policy suppose now, the EU subsidizes Airbus by 20 payoff units. The new game is to be found on the next page.

Strategic Trade Policy

The Nash equilibrium (P,NP) is unique; Airbus' dominant strategy is to produce. Compared to (NP,P) the welfare in the EU increases net by 100 units due to the usage of a strategic non-tariff barrier to trade.

		Boeing	
		P	NP
Air-	P	(10,−10)	(120,0)
bus	NP	(0,100)	(0,0)

Remark 1. G, F, and UK have born about one third of total R&D cost for the A 380 (some billion euro). In 2001 Boeing decided not to start with the 747x.

Remark 2. The same strategy in 2009 with regard to military transporters failed. The project was aborted (and restarted again at a later date).

Strategic Trade Policy

The deduced results depend essentially on the assumed companies' cost structure which have to be known by the respective government. Otherwise subsidies might be paid although they are not needed (→ free rider effect). Airbus' current problems initiated by the A 380 show also that a political direction of market decisions frequently induce unwanted consequences.

Last but not least we have to note counter-measures which may lead to a subsidy race having negative effects for tax payers in both countries (→ prisoners' dilemma, both produce). We would then need repeated games in mixed (i.e. randomized) strategies and imperfect information.

Excursion – Dumping

Subsidies for an export industry can lead to dumping, i.e. the export good is sold in the import country at a price which is lower than the corresponding price (→ *normal value*) in the export country.

- not plausible (→ transport cost)
- it is *legal* to sell below unit cost (if practiced at home)
- *GATT Art. VI.* Dumping is illegal if it causes material injury to the industry of an import country (→ anti-dumping measures).

Background

- Selling below unit cost as an activity of competition is per se legal because firms cannot hold this strategy too long (→ running the firm at a loss). A counter example for an illegal measure are cross subsidies (German Post: letter services subsidize parcel services)
- Regarding international trade especially the case of governmental subsidies is seen as an unfair practice.

Stylized overview

	"traditional" approach	"new" approach
market structure	perfect competition, homogeneous goods	oligopoly, monopolistic comp., heterogeneous goods
explanation of foreign trade	comparative advantages, inter-industrial trade	economies of scale, product differentiation, inter- and intra-industrial trade
specialization pattern	determined by preferences, technologies, factor endowments	indetermined or historically determined
effects of protectionism	small country: negative, big country: optimal tariff, terms of trade argument	trade policy can have positive or negative effects
in general	problem of counter-measures	

Contrasting Traditional and New Theory

Although the arguments in favor of free trade seem to be more ambiguous with regard to the "new" theory than for "traditional" theory, making use of protective measures embodies high risks.

- The theory based on imperfect competition is by far not homogeneous: numerous partial models with different implications.
- Scale effects in particular indicate that it is not useful to produce all goods (diversification) in a country.
- The problem of countervailing measures is independent of the market structure.

After all the infant industry argument has an exceptional position as the only economically convincing argument for protection.

Monopolistic Competition

The idea of monopolistic competition is used to explain intra-industrial trade in product variants which cannot be explained by the traditional approach based on perfect competition and homogeneous goods.

The market structure of monopolistic competition is defined as heterogeneous, bilateral polypoly.

- There are numerous producers and consumers.
- The goods are similar, but not identical (heterogeneous or differentiated goods).

The market is, therefore, imperfect. Monopolistic competition has characteristics of perfect competition (bilateral polypoly) as well as features of a monopoly (producers are monopolists for their own variant of a good). The easier (harder) the product variants can be substituted, the more monopolistic competition approximates perfect competition (monopoly).

Price Formation in a Monopoly

For the sake of understanding monopolistic competition, let us repeat the price formation in a Cournot monopoly. The revenue $r(x) = p(x)x$ refers to a price-demand function $p(x)$ indicating that a monopolist fixes the price rather than takes it as given. The monopolist either choses some quantity and accepts the resulting price or vice versa.

Profit maximization $\pi(x) = r(x) - c(x)$ gives the necessary condition

$$r'(x) = c'(x).$$

Opposite to perfect competition, where the demand curve seems to be horizontal at a given market price p, the monopolist now faces a decreasing demand curve $p(x)$.

Price Formation in a Monopoly

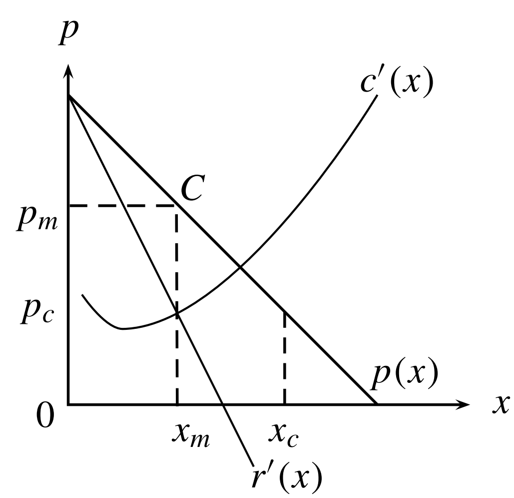
Example for a price-demand function $p(x) = c - dx$. Then revenue is

$$r(x) = cx - dx^2$$

and marginal revenue is

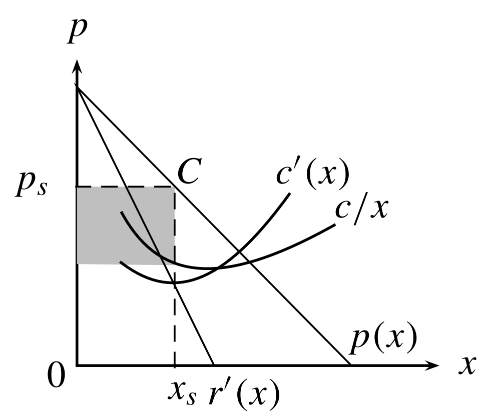
$$r'(x) = c - 2dx.$$

The marginal revenue curve is twice as steep as the demand curve. The point (x_m, p_m) on the demand curve is called Cournot's solution C.



Price Formation for Monopolistic Competition

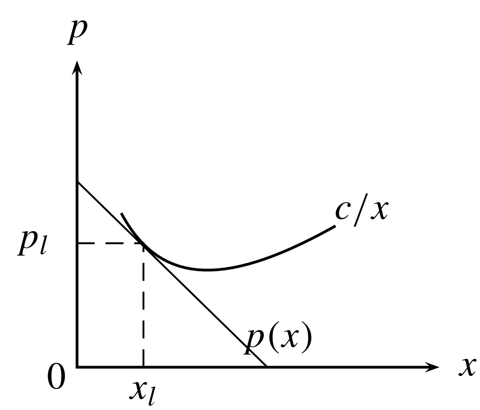
- In the short run the strategies of all other providers are given and the profit maximum is determined by $c'(x) = r'(x)$ as before.
- In fact, $r(x)$ depends on the strategies of all other competitors because individual demand results from total demand for all variants ($\rightarrow$ market share).
- The shaded area represents the profit of the monopolist at hand.
- As long as positive profits occur further producers enter the market.



4 International Trade under Imperfect Competition
4.3 Monopolistic Competition and Intra-Industry Trade

Price Formation for Monopolistic Competition

- Additional competitors reduce the market share of the monopolist at hand (shifts the individual demand curve downwards)
- In a long-term equilibrium the profits of all firms disappear ($p = c/x$).
- As no profit maximizing point (x_l, p_l) can lie above the average cost curve, the demand curve must ultimately tangent the c/x curve (long-term equilibrium). This is Chamberlin's tangent solution for monopolistic competition.



Price Formation for Monopolistic Competition

Similar to the monopoly we have with regard to monopolistic competition:

- A condition for profit maximization is $c'(x) = r'(x)$, where $r'(x) < p$.
- Consequently, the price exceeds marginal cost, $p > c'(x)$.

Similar to perfect competition we find with regard to monopolistic competition:

- In the long run the price equals unit cost, $p = c/x$.
- Additional competitors with additional variants reduce all profits to zero in the long-run equilibrium.

Price Formation for Monopolistic Competition

- Regarding perfect competition all firms produce at minimum long-run cost per unit ($\rightarrow$ firm's optimum), monopolistic competition reduces these outputs ($\rightarrow$ excess capacities).
- Regarding perfect competition prices equal the respective marginal cost (Pareto efficiency), monopolistic competition induces prices exceeding marginal cost (missing Pareto efficiency).

The increased variety of products, however, can improve consumers' welfare in comparison to the case of homogeneous goods. This fact is the basis for possible gains from intra-industrial trade with reference to monopolistic competition.

Krugman model

In accordance with Krugman (1979) we now present a simplified model of intra-industrial trade based on monopolistic competition.

Steps:

1. General equilibrium for a closed econonomy
   (a) demand for differentiated products
   (b) full employment on factor markets
   (c) supply of product variants (number of goods, quantities, price)
   (d) autarky solution
2. General equilibrium for open econonomies
   (a) modified equilibrium after transition to free trade
   (b) gains from trade

Krugman Model: Demand

(1.a) Demand for differentiated products

The utility of a representative consumer is a function of n differentiated goods.

$$U = \sum_{j=1}^{n} x_j^{\gamma}, \quad \text{with} \quad 0 < \gamma < 1$$

Utility maximization with regard to the budget constraint $y = \sum_{j=1}^{n} p_j x_j$ yields demand functions of the form ($\rightarrow$ Lagrangean approach)

$$x_j^D = \frac{y\, p_j^{1/(\gamma-1)}}{\sum_{k=1}^{n} p_k^{\gamma/(\gamma-1)}}$$

Krugman Model: Demand

Assuming a large number n of variants a change in one p_k has a negligible effect on the denominator of the demand function.

$$\sum_{k=1}^{n} p_k^{\gamma/(\gamma-1)} \approx \text{const.}$$

This implies a price elasticity of demand for good j as follows

$$\varepsilon := \frac{\partial x_j}{\partial p_j} \frac{p_j}{x_j} = \frac{1}{\gamma - 1} < -1$$

(We need this price elasticity to compute monopoly prices.)

Krugman Model: Factor Markets

(1.b) Full employment on factor markets

The inverse production function $v_j = a + bx_j$ indicates the quantity v_j of the *sole production factor labor* needed to produce x_j (a and b are positive parameters). For $x_j = 0$ we need the amount $v_j = a$ of labor (i.e., preparation requires time before production starts). This implies *increasing returns to scale* or decreasing unit cost.

The cost function results from multiplying the inverse production function by the wage rate q:

$$c(x_j) = qv_j = qa + qbx_j; \quad c'(x_j) = qb \qquad \forall j$$

Full employment requires

$$\text{(demand)} \;\; \sum_{j=1}^{n} v_j = \sum_{j=1}^{n} (a + bx_j) = v \;\; \text{(supply, const.)}$$

Krugman Model: Factor Markets

(1.c) Supply of product variants

Each firm maximizes its profit

$$\pi_j = p_j(x_j)x_j - qa - qbx_j,$$

so that

$$\frac{\partial \pi_j}{\partial x_j} = p_j + \frac{\partial p_j}{\partial x_j}x_j - qb = p_j \underbrace{\left(1 + \frac{\partial p_j}{\partial x_j}\frac{x_j}{p_j}\right)}_{=1+1/\varepsilon=\gamma} - qb = 0,$$

and thus

$$p_j = \frac{1}{\gamma}qb \quad \text{or} \quad \frac{q}{p_j} = \frac{\gamma}{b}$$

In a profit maximum the price p_j is given by some mark-up to marginal cost qb (note $1/\gamma > 1$). The real wage rate q/p_j is smaller than the marginal productivity of labor $1/b$ (note $\gamma < 1$).

Krugman Model: Consequences of Symmetry

As qb holds true for all firms, the above mark-up pricing indicates equal prices for all variants.

$$p_j = p = \frac{qb}{\gamma} \quad \text{for all} \quad j = 1, \ldots, n.$$

Substitution into the demand functions for goods shows that all quantities of goods must be equal in an equilibrium ($x_j^D = x_j^S = x$):

$$x_j = x = \frac{y}{np} \quad \text{for all} \quad j = 1, \ldots, n.$$

In what follows we can thus drop the index j and the utility function simplifies to

$$U = nx^\gamma$$

Long-term Autarky Equilibrium

(1.d) Long-term autarky solution

As long as positive profits exist further firms enter the market. In the long-run the profits of all firms must disappear for an equilibrium

$$\pi = 0 \iff px = c(x) = qa + qbx \iff x = qa/(p - qb)$$

Substituting $p = qb/\gamma$ into this condition we find the equilibrium quantity for all variants in the long run for a closed economy.

(I) $$x = \frac{a\gamma}{b(1-\gamma)}$$

This value together with the condition for full employment $v = n(a + bx)$ yields the number of variants or firms

(II) $$n = \frac{v(1-\gamma)}{a}$$

Foreign Trade Equilibrium

(2.a) Effects of a transition to free trade

Suppose for the sake of simplicity two countries of equal size starting free trade. This doubles the size of the global economy measured by the quantity of labor v.

- According to (II) we find that the number of variants n doubles (one variant per firm!).
- Eq. (I) states that the individual quantities, $x_j = x$, remain constant and so does the price $p_j = p$.
- The representative household consumes a doubled number of variants but one half of each quantity

$$x = \frac{y}{np}$$

Foreign Trade Equilibrium

(2.b) Gains from trade

- Each firms exports one half of its output and the rest is consumed at home. For constant outputs and prices the profit remains zero.
- The utility for open economies (i.e. U^f) increases compared to autarky (i.e. U^a) due to the effect that number of variants doubles but the houshold consumes one half of the former quantities.

$$U^f = 2n\left(\frac{x}{2}\right)^\gamma = 2^{1-\gamma} n x^\gamma = 2^{1-\gamma} U^a > U^a$$

- A similar argument holds good if both economies differ in size.

Foreign Trade Equilibrium

Remark. A constant output of each variant prevents the firms to take advantage of the assumed economies of scale.

Suppose therefore that the direct price elasticity of demand $\varepsilon < 0$ declines with n (i.e. $\varepsilon = \varepsilon(n)$ and $\varepsilon'(n) < 0$) because of increasing substitution possibilities. Due to $\gamma = 1 + 1/\varepsilon$ we find

$$\gamma'(n) > 0.$$

This results in the following effects (outline only):

- According to (II) the number of variants increases but it does not double.
- Following (I) the quantity of each variant grows. Due to decreasing unit cost this can be an additional source for gains from trade.

Summary

The Krugman model is able to explain intra-industrial trade on the basis of heterogeneous goods (variants of products) the number of which increases with the market size (cf. Ger $\rightarrow$ common market of the EU).

The transition from autarky to free trade increases welfare by two reasons: (a) increasing numbers of variants and (b) exploiting of scale economies.

Any barrier to trade would have negative effects in this model.

Total Differentials

The total differential of a differentiable function f of the variables x_1 and x_2 at the point $(\hat{x}_1, \hat{x}_2)$ describes (small) movements along a tangent plane.

$$y = f(\hat{x}_1, \hat{x}_2) \;\rightarrow\; \mathrm{d}y = \frac{\partial f(\hat{x}_1, \hat{x}_2)}{\partial x_1}\mathrm{d}x_1 + \frac{\partial f(\hat{x}_1, \hat{x}_2)}{\partial x_2}\mathrm{d}x_2$$

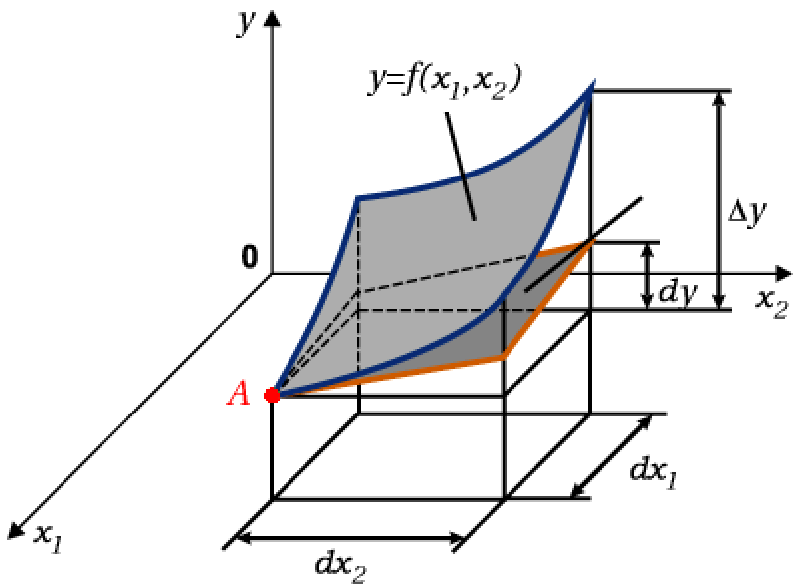
The implicit function theorem assumes $y =$ const. or $\mathrm{d}y = 0$

$$\frac{\mathrm{d}x_2}{\mathrm{d}x_1} = -\frac{\frac{\partial f(\hat{x}_1, \hat{x}_2)}{\partial x_1}}{\frac{\partial f(\hat{x}_1, \hat{x}_2)}{\partial x_2}}$$

This is used, e.g., to compute the slope of indifference curves ($\rightarrow$ MRS), isoquants ($\rightarrow$ MRS) or transformation curves ($\rightarrow$ MRT).

Total Differentials

total differential $$dy = \frac{\partial f(x_1, x_2)}{\partial x_1} dx_1 + \frac{\partial f(x_1, x_2)}{\partial x_2} dx_2$$



Envelope Theorem

Maximizing (or minimizing) a function $y = f(x; a)$ with respect to the variable x (a is a parameter) generates in general a solution $\hat{x}(a)$ and, therefore, $y(a) = f(\hat{x}(a); a)$. A parametric variation of a then yields

$$\frac{dy(a)}{da} = \underbrace{\frac{\partial f(\hat{x}(a); a)}{\partial x}}_{=0} \frac{d\hat{x}(a)}{da} + \frac{\partial f(\hat{x}(a); a)}{\partial a} \underbrace{\frac{da}{da}}_{=1} = \frac{\partial f(\hat{x}(a); a)}{\partial a}.$$

This observation is referred to as envelope theorem. Sloppy speaking it states that the optimal value $\hat{x}(a)$ may be seen as constant when differentiating $f(\hat{x}(a); a)$ with respect to the parameter a.

In economics this result is of special importance with regard to contrained problems (→ Lagrangean function).

Suppose the cost function c solves the following problem

$$c(q_1, q_2, x) = \min_{v_1, v_2} \{q_1 v_1 + q_2 v_2 | \; x \leq f(v_1, v_2)\}$$

The Lagrangean function

$$L = q_1 v_1 + q_2 v_2 + \lambda \left(x - f(v_1, v_2)\right)$$

helps to find factor demand functions v_1^D and v_2^D both depending on the parameters q_1, q_2 and x. A variation of one factor price leads to Shephard's lemma by making use of the envelope theorem:

$$\frac{\partial L(v_1^D, v_2^D, \hat{\lambda}; q_1, q_2, x)}{\partial q_i} = \frac{\partial c(q_1, q_2, x)}{\partial q_i} = v_i^D \qquad i = 1, 2$$

We simply treat v_1^D, v_2^D, $\hat{\lambda}$ as if they where constant.

Similar to Shephard's lemma, Hotelling's lemma refers to the profit function

$$\pi(p, q_1, q_2) = \max_{x, v_1, v_2} \{px - q_1 v_1 - q_2 v_2 | \; x \leq f(v_1, v_2)\}$$

Hotelling's lemma helps to compute the supply function

$$\frac{\partial \pi(p, q_1, q_2)}{\partial p} = x^S(p, q_1, q_2)$$

and correspondingly factor demand is given by

$$\frac{\partial \pi(p, q_1, q_2)}{\partial q_i} = v_i^D(p, q_1, q_2) \qquad \text{with } i = 1, 2$$

Homogeinity

If a production function f is homogeneous of degree r, the corresponding cost function c is homogeneous of degree $1/r$ in the output x.

$$\lambda^r f(v_1, v_2) = f(\lambda v_1, \lambda v_2) \quad \forall \lambda > 0 \implies \mu^{1/r} c(q_1, q_1, x) = c(q_1, q_2, \mu x) \quad \forall \mu > 0$$

Proof

$$c(q_1, q_2, \mu x) = \min\{q_1 v_1 + q_2 v_2 | \mu x = f(v_1, v_2)\}$$

Define $\lambda := \mu^{1/r}$ and set $v_i = \lambda \tilde{v}_i$ then

$$\begin{aligned} c(q_1, q_2, \mu x) &= \min\{q_1 \lambda \tilde{v}_1 + q_2 \lambda \tilde{v}_2 | \lambda^r x = f(\lambda \tilde{v}_1, \lambda \tilde{v}_2)\} \\ &= \lambda \min\{q_1 \tilde{v}_1 + q_2 \tilde{v}_2 | x = \lambda^{1/r} f(\lambda \tilde{v}_1, \lambda \tilde{v}_2)\} \\ &= \lambda \min\{q_1 \tilde{v}_1 + q_2 \tilde{v}_2 | x = f(\tilde{v}_1, \tilde{v}_2)\} \\ &= \mu^{1/r} c(q_1, q_2, x) \end{aligned}$$

Suppose $r = 1$ and set $\mu = 1/x$ then we find for the unit cost function

$$c(q_1, q_2, 1) = \frac{c(q_1, q_2, x)}{x} \quad \text{or} \quad c(q_1, q_2, x) = c(q_1, q_2, 1)\, x$$

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